Calculus II, Exam I, Fall 2013

Name:_____

Student signature:_____

Show all your work and give reasons for your answers.
Good luck!

Part I

Each problem in part I is worth **7** points; You must show your work to **justify your answers**!!

(1) Find the angle between the vectors $\vec{a} = < 1, 2, 1 >$ and $\vec{b} = < -1, 2, 2 >$. You can express your answer in arccos.

(2) Find the equation of the line perpendicular to the plane $3x - y + 2z = 4$ which passes through the point $(5, 4, 3)$.

(3) Find the area of the parallelogram spanned by the vectors $< -1, 0, 2 >$ and $< -2, -2, 1 >$.

(4) Find the distance from the point $(1, 2, 1)$ to the plane $x + 2y - 2z = 5$.

(5) Find the equation of the plane through the points $(1, 2, 1)$, $(-2, 1, 1)$ and $(-1, -2, 1)$.

(6) Find the point of intersection of the plane $x + 2y - z = 3$ and the line $\begin{cases} x = 1 + t \\ y = 1 - t \\ z = 2 + 2t \end{cases}$

(7) Are the vectors $\vec{a} =< 1, 2, 1 >$, $\vec{b} =< -2, 1, 2 >$ and $\vec{c} =< -3, 2, -1 >$ co-planar?

(8) If $\vec{r}(t) =< e^t, t^2, \sin(t) >$, find the unit tangent vector $\vec{T}(0)$.

Part II

(1) [**12 points**] Find the intersection of the planes $x + 2y - z = 3$ and $2x - y + z = 4$.

(2) **18 points**] Given the lines:

$$\ell_1 = \begin{cases} x = 1 + t \\ y = -1 + 2t \\ z = 3 + t \end{cases} \quad \text{and} \quad \ell_2 = \begin{cases} x = -1 - s \\ y = 2 + 3s \\ z = 1 + 2s \end{cases}$$

(a) If they intersect, find the point of intersection. If not, find their distance.

(b) Determine if they are skew or not.

(3) **[14 points]** Suppose that a particle's position at time t is given by $\vec{r}(t) =< \cos(t), \sin(t), t^3 >$. Find:
(a) The velocity vector $\vec{v}(\pi/3)$ at $t = \pi/3$.
(b) The equation of the tangent line to the graph of $\vec{r}(t)$ at $t = \pi/3$.