

Calculus II

Final Exam, Fall 2013

Name: _____

You must show all your work and give reasons for your answers. Good luck!

- (1) (5 points) Find the angle between the vectors $\langle 2, -1, 1 \rangle$ and $\langle 0, -1, 2 \rangle$. You may express your answer using $\arccos(\theta)$.

- (2) (5 points) Find the equation of the line perpendicular to the plane $2x + 3y - z = 5$ containing the point $(-1, 2, 1)$.

(3) (5 points) Evaluate $\int x \sin(x) dx$.

(4) (5 points) Evaluate $\int \frac{x^3+x}{\sqrt[3]{x}} dx$.

(5) (5 points) Evaluate $\int \ln(x) dx$.

(6) (5 points) Express $f(x) = \frac{x^2}{3+x}$ as a power series. **Also state the interval of convergence!**

(7) (5 points) Use series to approximate $\cos(\frac{1}{10})$ with an error less than 10^{-6} . [You do not need to compute and add the terms in the sum.]

(8) (5 points) Evaluate $\int \sin^3(x) \cos(x) dx$.

- (9) (5 points) Approximate $\int_0^{\frac{1}{10}} e^{-x^2} dx$ using a Riemann sum with $n = 3$ terms and the midpoint rule. **[You do not need to simplify or add the numbers!]**
- (10) (5 points) **Set up (but do not evaluate)** an integral for the volume of the solid of revolution obtained by rotating the region bounded by the curves $x = 1$, $x = 2$, $y = \frac{1}{x^2+1}$ and $y = -x - 1$ around the line $y = -3$.
- (11) (5 points) Compute the distance from the point $(1, 1, 1)$ to the plane $x + y + z = 1$.

(12) (10 points) Find the interval and radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{\sqrt{n}}$$

(13) (15 points) Evaluate

$$\int \frac{2}{(x+1)(x-1)^2} dx$$

- (14) (10 points) Find the work done in pumping all the water out of a full conical reservoir of height 5 m and radius 3 m ? [You may use the approximation $g \approx 10\text{ m/sec}^2$ and the density of water $\rho = 1000\text{ kg/m}^3$.]

- (15) (10 points) Use series to approximate $\int_0^{\frac{1}{10}} e^{-x^2} dx$ with an error less than 10^{-6} . (You do not need to compute and add the terms in the sum.)

Bonus Problem 4 points

We can approximate $\int_a^b f(x) dx$ in two ways. One by using Riemann sums (like in problem 9) and the other by using series (like problem 15).

- (a) What are the advantages and disadvantages in using the Riemann sum method?
- (b) What are the advantages and disadvantages in using the series method?