Calculus II, Final Exam, Spring 2013

Name:_____

You must show all your work and give reasons for your answers. Good luck!

(1) (5 points) Find the angle between the vectors $< -1, 2, 1 >$ and $< 0, -1, 2 >$. You may express your answer using $\arccos(x)$.

(2) (5 points) Find the equation of the line containing the points $(-1, 2, 1)$ and $(1, 2, -1)$.

(3) (5 points) Evaluate $\int \sqrt{x}(x^3+1)\,dx$.

(4) (5 points) Evaluate $\int \frac{x^2+x}{\sqrt[5]{x}}\,dx$.

(5) (5 points) Evaluate $\int \ln(x)\,dx$.

(6) (5 points) Express $f(x) = \frac{x^3}{4+x}$ as a power series. **Also state the interval of convergence!**

(7) (5 points) Use series to approximate $\sin(\frac{1}{10})$ with an error less than 10^{-6}. [You do not need to compute and add the terms in the sum.]

(8) (5 points) Evaluate $\int \sin(x)\, \cos^4(x)\, dx$.

(9) (5 points) Find the area of the region bounded by the curves $x = 0$, $x = 1$, $y = x^3 + 3$ and $y = -x^2 - x$.

(10) (5 points) **Set up (but do not evaluate)** an integral for the volume of the solid of revolution obtained by rotating the region bounded by the curves $x = 1$, $x = 2$, $y = \sin(x)$ and $y = -x - 1$ around the line $x = -3$.

(11) (5 points) **Set up (but do not evaluate)** an integral for the volume of the solid of revolution obtained by rotating the region bounded by the curves $x = 1$, $x = 2$, $y = \sin(x$ and $y = -x - 1$ around the line $y = -3$.

(12) (10 points) Find the interval and radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x+5)^n}{\sqrt[3]{n}}$$

(13) (15 points) Evaluate

$$\int \frac{4x}{(x-1)(x+1)^2}\,dx$$

(14) (10 points) Find the work done in pumping all the water out of a rectangular pool of $20 \times 40\ m$ which is $4\ m$ deep and which is half full (i.e. the water has to be lifted to the top of the pool before it can be removed). [You may use the approximation $g \approx 10\, m/sec^2$ and the density of water is $\rho = 1000\, kg/m^3$.]

(15) (10 points)Use series to approximate $\int_0^{\frac{1}{10}} \frac{x}{1+x^7}\,dx$ with an error less than 10^{-12}. (You do not need to compute and add the terms in the sum.)

Scratch paper