Name:_____

Student Number:_____

Exam I, Calculus III
Fall 2002

Show all your work; solutions must be justified. Attach additional pages if necessary.

(1) Let $\vec{\mathbf{r}}(t) =< t, 2t, t^2/2 >$.

(a) Find the the equation of the tangent line to the graph of $\vec{\mathbf{r}}(t)$ at the point $(1, 2, 1/2)$.

(b) Find the unit tangent vector $\vec{\mathbf{T}}(t)$ at the point $(1, 2, 1/2)$.

(c) Find the normal vector $\vec{\mathbf{N}}(t)$. NOTE: YOU DO NOT NEED TO SIMPLIFY THIS VECTOR NOR DO YOU NEED TO MAKE IT LENGTH 1.

(d) Find the curvature κ at the point $(1, 2, 1/2)$.

(e) If $\vec{\mathbf{r}}(t)$ specifies the position of a particle, find the velocity and acceleration vectors.

(f) Find the normal and tangential components of the acceleration vector.

(g) Express in words what the normal and tangential components of acceleration tell you about the movement of the particle.

(2) Given the unit tangent vector $\vec{\mathbf{T}}(t) =< t, t^2, t^4 >$ and the unit normal vector $\vec{\mathbf{N}}(t) = < \sin(t), cos(t), t >$ find the binormal vector $\vec{\mathbf{B}}(t)$.

(3) Find the limit or show it does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2 + 2xy}$$

(4) Find the linear approximation the the function $z = f(x, y) = \sin(x^2) + \cos(x^2) + \sqrt{x + y}$ at the point $(0, 0)$ and use it to approximate the value of $f(.1, -.1)$.

(5) If $z = \cos(xy) + y\cos(x)$, $x = u^2 + v$ and $y = u - v^2$ find $\frac{\partial z}{\partial u}$

(6) Find the maximal rate of change of the function $f(x, y) = x^2y + \sqrt{y}$ at the point $(2, 1)$ **and** the direction in which it occurs.

Find the equation of the tangent plane to the level to the sphere $x^2 + y^2 + z^2 = 2$ at the point $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$.