MA 227: Calculus III
Test #1, February 21, 2002

Time limit: 100 min.

Your name:

Your student ID:

1. Let $r(t) = \langle 2t^2, 3t - 1, t^3 \rangle$. Find a parametric equation for the tangent line to the above curve at the point $\langle 2, 2, 1 \rangle$.

10 points

2. Find the curvature of the space curve $r(t) = \langle t, t^2, t^3 \rangle$ at the point $\langle -1, 1, -1 \rangle$.

10 points

3. Let $r(t) = \langle t,\, t^2,\, 3t\rangle$. Find the tangential and normal components of the acceleration. (I. e. find a_T and a_N.)

10 points

4. Calculate the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2 \tan y}{x^2 + 0.5y^2}.$$

Explain your answer!

10 points

5. Use implicit differentiation to find $\partial z/\partial x$ and $\partial z/\partial y$.

$$x^2 + y^2 + z^2 = x(y + z).$$

10 points

6. Find the equation of the tangent plane to the surface $z = y \ln x$ at the point $\langle e, 1, 1\rangle$.

10 points

7. Let $u = xy + yz + zx$, $x = st$, $y = e^{st}$, $z = t^2$. Calculate $\partial u/\partial s$ and $\partial u/\partial t$ when $s = 0$, $t = 1$.

10 points

8. Let $f(x, y, z) = xy^2z^3$. Find the maximum rate of increase of f at the point $\langle 1, 1, 1\rangle$ and the direction in which it occurs. (Present this direction by the unit vector pointing to that direction.)

10 points