MA 227: Calculus III
Midterm Test #2, November 20, 2001

Time limit: 105 min.

Your name:

Your student ID:

1. Find $\partial z/\partial x$ and $\partial z/\partial y$ if

$$xy^2z^3 + x^3y^2z = x + y + z.$$

10 points

2. If $z = x^2 - xy + 3y^2$ and (x, y) changes from $(3, -1)$ to $2.96, -0.95$, compare the values of Δz and dz.

10 points

3. $u = xy + yz + zx$, $x = st$, $y = e^{st}$, $z = t^2$. Compute $\partial u/\partial s$ and $\partial u/\partial t$.

10 points

4. Find all directions u ($||u|| = 1$) in which the directional derivative of the function $f(x, y) = x^3 + xy + y^3$ at the point $(1, 0)$ is equal to 3.

10 points

5. Find the points on the surface $x^2y^2z = 1$ that are closest to the origin.

10 points

6. Find the maximum and minimum values of the function $f(x, y, z) = 2x - z$ subject to the condition $x^2 + 10y^2 + z^2 = 5$.

10 points

7. Evaluate the double integral by first identifying it as the volume of a solid.

$$\int\int_D \sqrt{1-x^2-y^2}dA,$$

where $D = \{(x,\, y)|\; x^2+y^2 \leq 1\}$.

10 points

8. Calculate the double integral

$$\int\int_R \frac{1+x^2}{1+y^2}dA,$$

where $R = \{(x,\, y)|\; 0 \leq x \leq 1,\; 0 \leq y \leq 1\}$.

10 points

9. Calculate the volume of the solid bounded by the cylinder $x^2+y^2 = 1$ and the sphere $x^2+y^2+z^2 = 9$.

10 points

10. Calculate the double integral

$$\int\int_R (x^2 + y^2)^2 dA,$$

where $R = \{(x,\, y) |\ 1 \le x^2 + y^2 \le 4\}$.

10 points