

Name: \_\_\_\_\_

Student Number: \_\_\_\_\_

**You must show your work and give reasons for your answers!**

**Good luck.**

1. Use Lagrange multipliers to find the maximum of the function  $f(x, y) = 9 - (x^2/4) - (y^2/4)$  subject to the constraint  $g(x, y) = x + 3y - 12 = 0$ .

2. The integral

$$\int_{-1}^1 \int_{x^2}^1 dy \, dx$$

represents the area of a region in the plane. Sketch the region and express the same area as a double integral with the order of integration reversed.

3. Find the centroid of the region in the first quadrant of the plane between the two circles of radius 1 and 2, and between the x-axis and the line  $y = x$ .

4. The volume of a solid is given by the triple integral:

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} dz \, dy \, dx.$$

Describe the solid (with a picture and by providing equations of its boundary). Express the integral in polar coordinates. [YOU DO NOT NEED TO EVALUATE THE INTEGRAL BUT YOU MUST STATE THE LIMITS OF INTEGRATION]

5. Find the mass of a solid semi-sphere of radius  $a$  whose density at any point is proportional to its distance from the center (of the entire sphere).
6. Find the center of mass and the moment of inertia about the y-axis of a thin plate bounded by the x-axis, the lines  $x = 1$  and  $x = -1$ , and the parabola  $y = x^2$  if the density  $\rho(x, y) = 7y + 1 \text{ kg/m}^2$ .