MA 227: Calculus III
Final Test, December 13, 2001

Time limit: 135 min.

Your name:

Your student ID:

1. The solid E in the first octant of space lies above the surface $z = \sqrt{3(x^2 + y^2)}$ and below the sphere $x^2 + y^2 + z^2 = 4$. Calculate its volume.

10 points

2. Evaluate

$$\int\int\int_E z dV,$$

where E lies between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in the first octant.

10 points

3. The lamina D is defined by the inequalities $0 \le x \le 1$, $0 \le y \le 1$, and its mass density function is given by $\rho(x, y) = x^3 + y^3$. Compute its mass and the center of mass.

10 points

4. The solid B lies inside the cylinder $x^2 + y^2 = 4$ and inside the ellipsoid $4x^2 + 4y^2 + z^2 = 64$. (And it is bounded by these surfaces.) Calculate its volume.

10 points

5. Evaluate the integral by reversing the order of integration.

$$\int_0^8 \int_{y^{1/3}}^2 e^{x^4} \, dx dy.$$

10 points

6. Calculate the integral

$$\int_1^4 \int_1^2 (\frac{x}{y} + \frac{y}{x}) dy dx.$$

10 points

7. Find the minimum and maximum values of the function $f(x, y, z) = yz + xy$ subject to the constraints $xy = 1$ and $y^2 + z^2 = 1$.

10 points

8. We know that x, y, and z are positive numbers the sum of which is equal to 100. Maximize the value of $x^a y^b z^c$. (a, b, and c are fixed (given) positive numbers; you cannot change them. You play the game with x, y, and z.)

10 points

9. Find the points on the hyperboloid $x^2 - y^2 + 2z^2 = 1$ where the normal line is parallel to the line connecting the points $(3, -1, 0)$ and $(13, 19, 30)$.

10 points

10. Let $z = f(x, y)$, $x = s + t$, $y = s - t$. Prove that

$$\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = \frac{\partial z}{\partial s} \cdot \frac{\partial z}{\partial t}.$$

10 points