Name:_____

Student Number:_____

You must show your work and give reasons for your answers!
Good luck.

(1) Given the equation $\mathbf{r}(t) =< \sin(t), t^3, \ln(t) >$:

(a) Find the equation of the tangent line to the graph of $\mathbf{r}(t)$ at the point $t = 1$.

(b) Find the curvature of the graph of $\mathbf{r}$ at the point $t = 1$

(c) What does the curvature tell you about the graph?

(2) Given the function $z = f(x, y) = x^3 + \sin(y)$,

(a) Find the equation of the tangent plane to the graph at the point $(1, 1)$

(b) Find the direction at the point $(1, 1)$ in which the function increases most rapidly. What is the rate of change in this direction?

(c) If $x = t^2$ and $y = t^3$, find $\frac{dz}{dt}$.

(3) Find the extreme values of the function $z = f(x, y) = x^2y$ in the region $x^2 + 2y^2 \leq 6$.

(4) Find the volume of the solid below the paraboloid $z = 3x^2 + y^2$ and above the region in the xy-plane bounded by $y = x$ and $x = y^2 - y$.

(5) A lamina occupies a region in the plane inside the circle $x^2 + y^2 = 2y$ but outside the circle $x^2 + y^2 = 1$. If the density at any point is **inversely** proportional to the distance from the origin, find its center of mass.

(6) Sketch the solid whose volume is given by the integral $\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dx\, dy\, dz$.

(7) State Stokes Theorem and the Divergence Theorem.

(8) Find $\oint_C F \cdot dr$, where $\mathbf{F}(x, y, z) =< yz, xz, xy >$ and C is the graph of the curve $\mathbf{r}(t) =< \sin^3(t^7)\cos(t), \cos^3(t^7)\sin(t), t^7 >$, $0 \leq t \leq 1$.

(9) Use green's Theorem to evaluate $\oint_C y^3 dx - x^3 dy$, where C is the positively oriented circle $x^2 + y^2 = 4$.