MA 227: Calculus III
Final Test, April 30, 2002

Timing: 4:15—6:45

Your name:

Your student ID:

1. If $z = x^2 - xy + 3y^2$ and (x, y) changes from $(3, -1)$ to $2.96, -0.95$, compare the values of Δz and dz.

10 points

2. If $z = f(x, y)$, where $x = s + t$, $y = s - t$, show that

$$\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = \frac{\partial z}{\partial s} \cdot \frac{\partial z}{\partial t}.$$

10 points

3. Find the maximum rate of change of $f(x, y, z) = x^2y^3z^4$ at the point $(1, 1, 1)$ and the direction in which it occurs.

10 points

4. Find the absolute minimum and maximum values of the function $f(x, y) = xy - x - y$ on the region bounded by the parabola $y = x^2$ and the line $y = 4$.

10 points

5. Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y, z) = 3x - y - 3z$ subject to the constraints $x + y - z = 0$ and $x^2 + 2z^2 = 1$.

10 points

6. Calculate the iterated integral

$$\int_0^{\pi/2} \int_0^{\pi/2} \sin(x + y) dy dx.$$

10 points

7. Evaluate the double integral

$$\iint_D x\sqrt{y^2 - x^2}\,dA$$

on the region

$$D = \{(x, y)|\ 0 \leq y \leq 1, \quad 0 \leq x \leq y\}\,.$$

10 points

8. Use polar coordinates to compute the volume of the solid bounded by the paraboloid $z = 10 - 3x^2 - 3y^2$ and the plane $z = 4$.

10 points

9. A lamina D occupies the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant, and the mass density function is $\rho(x,\, y) = 2(x^2 + y^2)$. Calculate the moments of inertia I_x, I_y, and I_0.

10 points

10. Evaluate the triple integral

$$\iiint_E yz\cos(x^5)dV,$$

where

$$E = \left\{(x, y, z)\big|\; 0 \leq x \leq 1,\; 0 \leq y \leq x,\; x \leq z \leq 2x\right\}.$$

10 points