MA 227, Fall 2001

TEST 1

Duration 105min; Max. Points 44

Make sure to show all your work and underline the final results of each problem. Write your name on this sheet and use it as a cover page when you turn in your work. Do not write your results or computations on this paper. Good luck!

1. (4 pts.) Find domain of the vector function

$$\mathbf{r}(t) = \langle \sqrt{2-t}, 1/t, \ln(t) \rangle$$

Explain your answer.

2. (6 pts.) Let $\mathbf{r}(t) = \langle \sqrt{1+t^2}, t \rangle$

(a) Sketch the curve given by $\mathbf{r}(t)$.

(b) Compute $\mathbf{r}'(t)$.

(c) Give a parametrization of the tangent line to the curve through the point $(\sqrt{2}, 1)$.

3. (6 pts.) Evaluate the integrals

$$(a) \quad \int_1^4 (\sqrt{t}\mathbf{i} + te^{-t}\mathbf{j} + t^{-2}\mathbf{k})\, dt$$

$$(b) \quad \int \langle \cos(t), 1/t, \frac{1}{1+t^2} \rangle\, dt.$$

4. (4 pts.) Find the length of the curve from $(4, 4, 0)$ to $(0, 0, 3\pi/2)$ given by the vector function $\mathbf{r}(t) = \langle 4\cos(t), 4\cos(t), 3t \rangle$.

5. (4 pts.) Which of the formulas:

$$(i)\ \kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}, \qquad (ii)\ \kappa(t) = \frac{|(\dot{x}\ddot{y} - \ddot{x}\dot{y})^{3/2}|}{(\dot{x}^2 + \dot{y}^2)^{3/2}}, \qquad (iii)\ \kappa(t) = \frac{|f''(x)|}{(1 + f'(x)^2)^{3/2}}$$

to can be used to compute the curvature of the curve described by the given equation(s). Do NOT compute the curvature.

(a) $\mathbf{r}(t) = \sin(t)\mathbf{i} + \cos(2t)\mathbf{j} + e^t\mathbf{k}$.

(b) $u(t) = \dfrac{\sin(t)}{t}, \qquad v(t) = t.$

6. (6 pts.) Sketch a contour map of the function

(a) $f(x, y) = 4x^2 + y^2$

(b) $f(x, y) = xy^2$

7. (6 pts.) Find the limit or show that it does not exists

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x|y|^{1/2}}{x^2 + |y|}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{1 + xy}{e^x(1 + \cos y)}$

8. (4 pts.) Find the first partial derivatives

$$z(x, y) = \frac{x - y}{x + y}$$

9. (4 pts.) The function $z(x, y)$ is implicitly defined by $-\cos(xy) = xe^{yz}$. Compute $\partial z/\partial x$.