

Math 227 Test #1, January 23, 2002
Show all of your work.

1. (10 pts.) Sketch the curve $\mathbf{r}(t) = (\cos t, \sin t, t)$. Indicate with an arrow the direction in which t increases.

2. (10 pts.) Use the definition

$$\mathbf{r}'(0) = \lim_{t \rightarrow 0} \frac{\mathbf{r}(t) - \mathbf{r}(0)}{t}$$

to find the derivative of $\mathbf{r}(t) = t\mathbf{i} - \sin t\mathbf{j} + t^2\mathbf{k}$ at $t = 0$.

3. (10 pt.) Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = \cos(\pi t)\mathbf{i} + \sin(\pi t)\mathbf{j} - 2t\mathbf{k}$ and $\mathbf{r}(0) = \mathbf{k}$.

4. (24 pts) Consider the curve $\mathbf{r}(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + t \mathbf{k}$.

(a) Find the arc length of the curve for $0 \leq t \leq 2$.

(b) Find parametric equations of the tangent line of the curve at $(1, 0, 0)$.

(c) Find an equation of the normal plane of the curve at $(1, 0, 0)$.

(d) Find the curvature function $\kappa(t)$.

5. (12 pt.) For $\mathbf{r}(t) = \mathbf{i} + t\mathbf{j} + t^2\mathbf{k}$, find the velocity, the speed and the acceleration. What are the tangential and normal components of the acceleration?

6. (10 pt.) Find and sketch the domain of $f(x, y) = \sqrt{36 - 9x^2 - 4y^2}$. What is the range?

7. (12 pts) Find the limit (explain why)

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xyz}{x^2 + y^2 + z^2}.$$

8. (12 pts) Find the largest set on which the function

$$f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

is continuous (explain why).