Math 227 Test #2, February 18, 2002
Show all of your work.

1. (15 pt.) Let $f(x, y, z) = x^2 + y^2 + z^2 + xyz$.

   (a) Find $\partial f/\partial x$, $\partial f/\partial y$ and $\partial f/\partial z$.

   (b) Find $f_x(3, 4, 0)$, $f_y(1, 0, 0)$ and $f_z(1, 0, 0)$.

   (c) Find $\frac{\partial^2 f}{\partial y \partial z}$ and $\frac{\partial^3 f}{\partial y^2 \partial x}$.

2. (20 pt.) Let $f(x,y) = \sqrt{x^2+y^2}$.

(a) Find an equation of the tangent plane to the surface $z = f(x,y)$ at the point $(3,4,5)$.

(b) Find the linearization of $f(x,y)$ at $(3,4)$ and use it to approximate $f(2.95, 4.08)$.

(c) Find the differential of $z = f(x,y)$.

(d) If $z = f(x,y)$ and (x,y) changes from $(3,4)$ to $(2.95, 4.08)$, compare Δz and dz.

3. (10 pt.) Let $f(x, y, z) = xe^y + yz + ze^x$.

   (a) Find $\partial f/\partial s$ and $\partial f/\partial t$ if $x = s + t$, $y = s - t$ and $z = st$.

   (b) Find $\partial z/\partial x$ and $\partial z/\partial y$ for $f(x, y, z) = 0$.

4. (10 pt.) Consider the surface $x^2y^2z = 1$.

   (a) Find an equation of the tangent plane of the surface at $(1, 1, 1)$.

   (b) Find an equation of the normal line of the surface at $(1, 1, 1)$.

5. (10 pt.) The temperature at a point (x, y, z) is given by $T(x, y, z) = 200e^{-x^2-3y^2-9z^2}$. In which direction does the temperature increase the fastest at the point $(1, 1, 1)$? What is the maximum rate of increase?

6. (15 pts) Find the local maximum and minimum values and saddle point(s) of the function $f(x, y) = 12 - x^2 + 4x - 2y^2 - 8y$.

7. (20 pt.) A cardboard box without a lid is to have volume 32 cm^3.

(a) Use direct minimization to find the dimensions that minimize the amount of cardboard used.

(b) Use the method of Lagrange Multipliers to redo part (a) (verification).