MA 227, Fall 2001.

FINAL EXAM

Duration 140 min, Max. Points: 32.

Make sure to show all your work and underline your final results. Write on these sheets or use extra paper if needed. Each problem is worth 4 points. Good luck!

1. Find dy/dx if $\cos(x-y) = xe^y$.

2. Find the maximum rate of change of the function $f(x,y)$ at $(1,1)$ and the direction in which it occurs.

$$f(x,y) = x^2 - \frac{x}{y^3}$$

3. Find the absolute maximum and minimum values of the function

$$f(x, y) = e^{-x^2-y^2}(x^2 + 2y^2)$$

on the disk $x^2 + y^2 \leq 4$.

4. Use polar coordinates to evaluate

$$\int_0^{\sqrt{2}} dy \int_y^{\sqrt{4-y^2}} dx \frac{1}{1+x^2+y^2}.$$

5. Set up a triple integral for the volume of the region below the plane $z = x+y$ and above the domain D in the xy-plane bounded by the parabolas $y = x^2$ and $x = y^2$. Evaluate this integral.

6. Find the work done by the force field $\mathbf{F}(x, y) = \langle x, y + 2 \rangle$ in moving an object along an arch of the cycloid $\mathbf{r}(t) = \langle t - \sin(t), 1 - \cos(t) \rangle$, where $0 \le t \le 2\pi$.

7. Determine whether or not $\mathbf{F}$ is a gradient field

(a) $\mathbf{F}(x, y) = \langle 2x\sin(y), y^3/5 + x^2\cos(y)\rangle$

(b) $\mathbf{F}(x, y) = -3x^2y\mathbf{i} + (x-y)^2\mathbf{j}$

8. Use the formula $\text{area}(D) = \int_{\partial D} x\,dy$ to show that the area enclosed by an ellipse with semi axis a and b is $ab\pi$. To do this you need a parametrization of the ellipse $x^2/a^2 + y^2/b^2 = 1$.