

**Math 227 Test #3,** March 18, 2002  
Show all of your work for full credit.

1. (20 pt.) Let  $R = [0, 1] \times [0, 1]$ .

(a). Use the Midpoint Rule to estimate the value of  $\int \int_R 12x^2y^3 dx dy$  with  $n = m = 2$ .

(b). Evaluate  $\int \int_R 12x^2y^3 dx dy$ .

(c). Evaluate  $\int \int_R (8y - 3x^2) dx dy$ .

2. (20 pt.) Evaluate the following double integrals.

(a).  $\int \int_D 2\sqrt{x}y dx dy$  where  $D = \{(x, y) | 0 \leq x \leq 1, -x \leq y \leq \sqrt{x}\}$ .

(b).  $\int \int_D xy^3 dx dy$  where  $D = \{(x, y) | 0 \leq y \leq 1, 1 - y \leq x \leq 1\}$ .

(c).  $\int \int_D e^x dx dy$  where  $D$  is the triangular region with vertices  $(0, 0)$ ,  $(2, 4)$  and  $(6, 0)$ .

3. (20 pt.) Find the volume of the solid under the paraboloid  $z = x^2 + y^2$  and above the region bounded by  $y = x^2$  and  $x = y^2$ .

4. (20 pt.) A lamina occupies the region inside the circle  $x^2 + y^2 = 2y$  but outside the circle  $x^2 + y^2 = 1$ . Find the center of mass if the density at any point is inversely proportional to its distance from the origin.

5. (20 pt.) A lamina occupies the region bounded by the parabola  $x = y^2$  and the line  $y = x - 2$ . Find the moments of inertia  $I_x$ ,  $I_y$  and  $I_0$  if the density  $\rho(x, y) = 3x$ . Is it more difficult to rotate the lamina about the  $x$ -axis or  $y$ -axis?