Math 227 Test #4, April 19, 2002

Show all of your work for full credit.

1. (20 pt.) Evaluate the following integrals.

(a). $\int_0^1 \int_0^z \int_0^x 6xzdydxdz$.

(b). $\int\int\int_E yz\cos(x^5)dxdydz$ where $E = \{(x, y, z)|0 \leq x \leq 1, 0 \leq y \leq x, 0 \leq z \leq y\}$.

(c). $\int\int\int_E xdxdydz$ where E is the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.

2. (20 pt.) Use a triple integral to find the volume of the solid E bounded by the paraboloid $z = 2 - x^2 - y^2$ and the cone $z = \sqrt{x^2 + y^2}$.

3. (20 pt.) Show that the line integral $\int_C (2x \sin y + 4x^3)dx + (x^2 \cos y - 3y^2)dy$ is independent of path and evaluate the integral, where C is any path from $(1, 1)$ to $(3, 4)$.

4. (20 pt.) A wire is bent into the shape of the helix $x = t$, $y = \cos t$, $z = \sin t$, $0 \leq t \leq 2\pi$.

(a). Find its mass if its density at any point is proportional to the square of its distance from the origin.

(b). Find the work done by the force field $\mathbf{F}(x, y, z) = yz\mathbf{i} - z\mathbf{j} + y\mathbf{k}$ on a particle that moves along the wire from $t = 0$ to $t = 2\pi$.

5. (20 pt.) Let H be the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$, whose density at any point is proportional to its distance from the origin. Find the center of mass of H.