Math 227 FINAL, April 29, 2002

Show all of your work for full credit.

1. (20 pt.) Let $\mathbf{r}'(t) = \cos t\mathbf{i} + \mathbf{j} - \sin t\mathbf{k}$ and $\mathbf{r}(0) = \mathbf{k}$.

   (a) Find $\mathbf{r}(t)$.

   (b) Find parametric equations of the tangent line of the curve $\mathbf{r}(t)$ at $(0, 0, 1)$.

   (c) Find an equation of the normal plane of the curve $\mathbf{r}(t)$ at $(0, 0, 1)$.

   (d) Find the curvature function $\kappa(t)$ of the curve $\mathbf{r}(t)$.

2. (20 pt.) Let $f(x,y) = \frac{x^3+y^3}{x^2+y^2}$. Do the following and justify your answers.

(a) Find and sketch the domain of $f(x,y)$.

(b) Find the limit

$$\lim_{(x,y)\to(0,0)} f(x,y) = .$$

(c) Find the largest set on which the function $f(x,y)$ is continuous.

(d) Define $f(x,y)$ on the whole plane so that it is continuous.

3. (20 pt.) Let $f(x, y) = \sqrt{40 - x^2 - 3y^2}$.

(a) Find an equation of the tangent plane to the surface $z = f(x, y)$ at the point $(1, 1, 6)$.

(b) Find an equation of the normal line to the surface $z = f(x, y)$ at the point $(1, 1, 6)$.

(c) Use the linearization of $f(x, y)$ at $(1, 1)$ to approximate $f(.95, 1.08)$.

(d) If $z = f(x, y)$ and (x, y) changes from $(1, 1)$ to $(.95, 1.08)$, compare Δz and dz.

4. (20 pt.) Let $f(x, y, z) = xe^{2y} + \sin(yz) + z \ln x$.

(a) Find $\partial f/\partial s$ and $\partial f/\partial t$ if $x = s + t^2$, $y = s^3 - t$ and $z = 2st$.

(b) Find $\partial z/\partial x$ and $\partial z/\partial y$ for $f(x, y, z) = 0$.

(c) Find the direction derivative of $f(x, y, z)$ at $(1, 1, 1)$ in the direction $\mathbf{u} = (1, 1, 1)$.

(d) In which direction does the direction derivative of $f(x, y, z)$ at the point $(1, 1, 1)$ have a maximum value and what is the maximum value?

5. (20 pt.) A rectangular box is to have total surface area 64 cm^2.

(a) Use direct minimization to find the dimensions that maximize the box's volume.

(b) Use the method of Lagrange Multipliers to redo part (a) (verification).

6. (20 pt.) Evaluate the following integrals.

(a) $\int\int_D y dx dy$ where D is the region bounded by $y = x^3$ and $x = y^2$.

(b) $\int\int\int_E y dx dy dz$ where E is the solid under the paraboloid $z = 4 - x^2 - y^2$ and above the region bounded by $y = x^2$ and $y = x$.

7. (20 pt.) Let E be the solid bounded by the paraboloid $z = 8 - x^2 - y^2$ and the cone $z = -2\sqrt{x^2 + y^2}$.

(a) Use a double integral to find the volume of E.

(b) Use a triple integral to redo (a).

8. (20 pt.) A lamina occupies the region inside the circle $x^2 + y^2 = 2x$ but outside the circle $x^2 + y^2 = 1$. Find the moments of inertia I_x, I_y and I_0 if the density $\rho(x, y) = 5x$.

9. (20 pt.) Let H be the solid that lies above the cone $z = \sqrt{3x^2 + 3y^2}$ and below the sphere $x^2 + y^2 + z^2 = 2z$, whose density at any point is inversely proportional to the square of its distance from the origin. Find the center of mass of H.

10. (20 pt.) Consider the semicircle C: $x^2 + y^2 = 1$, $y \geq 0$ and $y = 0$, $-1 \leq x \leq 1$.

(a) Find the work done by the force field $\mathbf{F}(x, y) = x^2\mathbf{i} + xy\mathbf{j}$ on a particle that moves once around C in the counterclockwise direction by directly evaluating a line integral.

(b) Evaluate the line integral of work in (a) by using Green's theorem.