Calculus III Test 1 Jan. 31, 2002 NAME_____

No calculators, books, or notes allowed. Justify your answers by giving appropriate arguments and steps. Circle answers. All problems will be of equal value. Be sure to work the given problem; otherwise you will not receive credit.

1. Find parametric equations for the curve formed by the intersection of the plane $2x + 3y + z = 1$ with the cylinder $x^2 + y^2 = 4$.

2. Let the curve C be given by $x = t$, $y = -1 + t^2$, $z = 1 + t^3$. Find parametric equations for the tangent line to the curve at the point $(1, 0, 2)$.

3. Find the length of the curve $\vec{\mathbf{r}}(t) = (\cos t)\mathbf{i}+(\sin t)\mathbf{j}+2t^{3/2}\mathbf{k}$ for $0 \leq t \leq 7$.

4. Consider an ellipse $x^2/a^2 + y^2/b^2 = 1$ $(a, b > 0)$. Find its curvature at the vertices $(a, 0)$ and $(0, b)$. (Hint: Try letting $x = a\cos t$, $y = b\sin t$.)

5. Let $\vec{\mathbf{r}}(t) = (\sin 3t)\mathbf{i} + (\cos 3t)\mathbf{j} + t\mathbf{k}$. Reparameterize the curve with respect to arc length s measured from the point where $t = 0$, in the direction of increasing t.

6. Find the unit tangent, unit normal, and unit binormal vectors $\vec{\mathbf{T}}(t), \vec{\mathbf{N}}(t)$, and $\vec{\mathbf{B}}(t)$ for the curve given by $x = t^2$, $y = \ln t$, $z = 2t$.

7. Find the velocity, speed, and acceleration at time t for a particle with position given by $\overrightarrow{\mathbf{r}}(t) = (\cos 2t)\mathbf{i} - (\sin 2t)\mathbf{j} + t\mathbf{k}$.

8. A projectile is fired from ground level with an initial speed of $1000m/s$ at a 60° angle of elevation above the horizontal. Ignoring friction, find the range of the projectile. (Recall downward acceleration due to gravity is $-9.8m/s^2$).

9. Find a parametric representation for the surface of the cylinder $x^2 + z^2 = 16$.

10. Sketch and label at least five level curves of the function $f(x, y) = x/(2 + y^2)$.

11. Suppose a space ship's position is given at time $t \geq 0$ by $\overrightarrow{\mathbf{r}}(t) = t\mathbf{i} + 3t^2\mathbf{j}$, and the coordinates of a space station are $(4, 36)$. When should the engines be turned off so that the space ship can coast into the space station?

Extra Credit: Find the angle at which the two spirals $\overrightarrow{\mathbf{r}}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$ and $\overrightarrow{\mathbf{s}}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + 2t\mathbf{k}$ intersect.