

Calculus III Test 2 Feb. 28, 2002 NAME_____

No calculators, books, or notes allowed. Justify your answers by giving appropriate arguments and steps. Circle answers. Unless stated otherwise, a problem is worth 10 points. Be sure to work the given problem; otherwise you will not receive credit.

1. (15 pts) Let $f(x, y) = x^2 + 3y^2 + 3xy + 5x^2y^3$. Find:

(A) f_x

(B) f_y

(C) f_{xy}

2. Let g be a twice differentiable function of a single variable. Find all values of c so that $u(x, t) = g(x - ct)$ is a solution of the wave equation $u_{tt} = 3u_{xx}$.

3. Find an equation for the plane tangent to $z = x/y$ at the point $(x, y, z) = (6, 2, 3)$.

4. Suppose $x^2 + y^2 + z^2 - xyz = 1$. Find $\partial z / \partial x = z_x$.

5. The dimensions of a closed rectangular box are measured as 20cm, 30cm, and 40cm, with a possible error of 0.1cm in each dimension. Use differentials to estimate the maximum error in calculating the volume of the box.

6. Let $f(x, y) = \ln|3x + 5y|$. Find the directional derivative of f at the point $(1, -2)$ in the direction toward the origin.

7. Let $w = \frac{x}{y} + \frac{y}{z}$. Find the maximum rate of change of w at $(x, y, z) = (2, 1, -1)$ and the direction in which it occurs.

8. Find an equation of the plane tangent to the ellipsoidal surface $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 3$ at the point $(x, y, z) = (1, 3, -2)$.

9. Let $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$. Find all critical points and identify all local minima, maxima, and saddle points.

10. Use the method of Lagrange multipliers to find the minimum and maximum values of $f(x, y, z) = 2x + 3y - 4z$ on the sphere $x^2 + y^2 + z^2 = 1$.

11. Use any valid method to find the area of the largest rectangle which can be inscribed in the general ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

EXTRA CREDIT: Let $g(x, y) = (x^2 - 1)^2 + (x^2y - x - 1)^2$. Find all critical points, and identify each as a point of local minimum, maximum, or saddle. Also find the absolute minimum value of g .