

Calculus III Test 3 April 11, 2002 NAME _____
STUDENT NUMBER: _____

No calculators, books, or notes allowed. Justify your answers by giving appropriate arguments and steps. Circle answers. All problems will be of equal value. Be sure to work the given problem; otherwise you will not receive credit. Use your own paper, write on ONE SIDE ONLY of each sheet, number pages, put your name on every page.

1. Evaluate $\int_1^2 (xy - x^2) dy$
2. Evaluate $\int_2^3 \int_0^1 (x + y)^{-2} dx dy$
3. Evaluate $\iint_D xy dA$ where D is the triangular region with vertices $(0,0)$, $(0,2)$, and $(3,0)$.
4. Find the volume of the solid under $z = x + 2y$ which is above the region bounded by $y = 2x$ and $y = x^2$.
5. Sketch the region of integration and change the order of integration for $\int_0^2 \int_{x^2}^{2x} f(x, y) dy dx$.
6. Use polar coordinates to find the volume of the solid under $z = 1 + x^2 + y^2$ and above the ring $4 \leq x^2 + y^2 \leq 9$.
7. Find the centroid of the region bounded by $y = x$ and $y = x^2$.
8. Write a triple iterated integral for the volume of the solid bounded by the cylinder $4x^2 + z^2 = 4$ and the planes $y = 0$ and $y = z + 2$. Do not evaluate.
9. Use spherical coordinates to find the volume of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 4$.

Extra Credit: Let D be the solid bounded by the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

Find the volume of D by changing variables in the triple integral $\iiint_D dV$.
(Let $x = au$, $y = bv$, $z = cw$).