Calculus III Final Exam. April 2002. Name_____
READ: Mathematically justify your answers (show work). Simplify and complete all computations as much as possible. Circle answers. No calculators, books, or notes allowed.

If you would like to have your final grade posted on my office door (by next Monday...don't telephone) give a code name to use:_____

1. Let the curve C be given by $x = t^2$, $y = 1-t^3$, $z = 1+3t$. Find parametric equations for the tangent line to the curve at the point $(x, y, z) = (1, 0, 4)$.

2. Find the arc length of the curve given by $x = t^2$, $y = \frac{\sqrt{5}}{3}t^3$, for $0 \leq t \leq 1$. (This can and should be calculated exactly!)

3. A projectile is fired from ground level with an initial speed of 98m/s at a 45° angle of elevation above the horizontal. Ignoring friction, find (A) the position $\vec{r}(t)$ at time t (before it strikes the ground), and (B) the range of the projectile.

4. Let $f(x, y) = x^4 + y^5 + xy^3 + x^2y$. Find (and simplify)
(A) f_x

(B) f_y

(C) f_{xy}

5. Find an equation for the plane tangent to the surface $xy^2z^3 = 1$ at the point $(x, y, z,) = (\frac{1}{4}, 2, 1)$.

6. Evaluate $\int_2^3 \int_0^1 (x + y)^{-3} dx\, dy$

7. Let $f(x, y) = 2y^3 + x^2y + 5y^2 + x^2$.

(A) Find all critical points.

(B)Identify all local minima, maxima, and saddle points.

8. Use the method of Lagrange multipliers to find the minimum and maximum values of $f(x, y, z) = 2x - y + 3z$ on the ellipsoidal surface $x^2 + \frac{y^2}{3} + z^2 = 1$.

9. Evaluate $\iint_D xydA$ where D is the triangular region with vertices $(0, 0), (0, 4)$, and $(1, 3)$.

10. Find the volume of the solid under $z = x + y$ and above that portion of the ring $1 \leq x^2 + y^2 \leq 9$ which lies in the first quadrant.

11. Let the curve C be given by $\overrightarrow{r}(t) = \cos(t)\overrightarrow{i} + \sin(t)\overrightarrow{j}$ for $0 \leq t \leq \frac{\pi}{2}$. Evaluate $\int_C ydx - xdy$.

12. Suppose the force exerted on a unit mass at the point (x, y) is $\overrightarrow{F}(x, y) = (2x + y)\overrightarrow{i} + (x + 2y)\overrightarrow{j}$.

(A) Show the field is conservative and find its potential.

(B) Find the work done in moving the mass from the origin along the $x-$axis to $(3, 0)$ and then vertically to $(3, 5)$.

13. Let C be a simple closed curve, traversed counterclockwise, and let D denote the region inside C. Use Green's theorem to show that $\oint_C (x^2y^3 + 2y)dx + (x^3y^2 + 5x)dy$ depends only on the area of D, and find how it depends on the area of D.

14. Suppose a space ship's position is given at time $t \geq 0$ by $\overrightarrow{r}(t) = t\overrightarrow{i} + \frac{1}{3}t^2\overrightarrow{j}$, and the coordinates of a space station are $(5, 7)$. When should the engines be turned off so that the space ship can coast into the space station?

Surprise Extra Credit: For ideal gasses, pressure P, volume V, and temperature T satisfy the law $PV = T$. Show that

$\frac{\partial P}{\partial V} \cdot \frac{\partial V}{\partial T} \cdot \frac{\partial T}{\partial P} = -1$. (Hint: In each partial derivative, one assumes one variable is dependent and the other two are independent.)