FALL 2006 — MA 227-8B — FINAL

Name: ____

1. Part I

There are 10 problems in Part I, each worth 4 points. Place your answer on the line below the question. In Part I, there is no need to show your work, since only your answer on the answer line will be graded.

(1) Compute the cross product of the vectors $\langle 0, -1, 1\rangle$ and $\langle 2, 1, 3\rangle$.

Answer: $\langle -4, 2, 2\rangle$.

(2) Find the indefinite integral $\int (3\,\mathbf{i} - 2t\,\mathbf{j} + \frac{1}{t}\,\mathbf{k})\,dt$.

Answer: $\langle 3t, -t^2, \ln t\rangle + \mathbf{C}$.

(3) Find the curvature of the graph of the function $y = \cos 2x$ at the point $(0, 1)$.

Answer: 4.

(4) Describe or sketch the domain of the function $f(x, y) = \sqrt{x^2 + y^2 - 1}$.

Answer: $x^2 + y^2 \geq 1$.

(5) Find the linearization $L(x,y)$ of $f(x,y) = \ln(x+y^2)$ at the point $(1,1)$.

Answer: $L(x,y) = \ln 2 + \frac{1}{2}x + y - \frac{3}{2}$.

(6) Evaluate $\int_{-1}^{0} \int_{0}^{2} 2xy\, dy\, dx$.

Answer: -2.

(7) Reverse the order of integration in the iterated integral $\int_0^1 \int_{x^2}^1 f(x,y)\, dy\, dx$. [Do not compute the integral!]

Answer: $\int_0^4 \int_0^{\sqrt{y}} f(x,y)\, dy\, dx$.

(8) Compute the Jacobian of the transformation $x = u^2 - v$, $y = u + v^2$.

Answer: $4uv + 1$.

(9) Find the divergence of the vector field $\mathbf{F}(x,y,z) = \langle\, x^2 \sin z\,,\, xy\,,\, x\cos z\,\rangle$.

Answer: $x \sin z + x$.

(10) Find the potential function of the following conservative vector field:

$$\mathbf{F} = ye^{xy}\,\mathbf{i} \,+\, (1 + xe^{xy})\,\mathbf{j}$$

Answer: $f(x,y) = e^{xy} + y + C$.

2. Part II

There are 5 problems in Part II, each worth 12 points. On Part II problems show all your work! Your work, as well as the answer, will be graded. Your solution must include enough detail to justify any conclusions you reach in answering the question.

(1) The position vector of a moving particle is given by $\mathbf{r}(t) = 4\sin t\,\mathbf{i} - 4\cos t\,\mathbf{j} + 3t\,\mathbf{k}$.
  (a) Find the particle's velocity $\mathbf{v}$ and acceleration $\mathbf{a}$.
  (b) Find the unit tangent vector $\mathbf{T}$ and the unit normal vector $\mathbf{N}$.

Answers:

$$\mathbf{v}(t) = \langle 4\cos t, 4\sin t, 3\rangle$$

$$\mathbf{a}(t) = \langle -4\sin t, 4\cos t, 0\rangle$$

$$\mathbf{T}(t) = \tfrac{1}{5}\langle 4\cos t, 4\sin t, 3\rangle$$

$$\mathbf{N}(t) = \langle -\sin t, \cos t, 0\rangle$$

(2) Suppose that over a certain region of space the electrical potential V is given by

$$V(x, y, z) = x^2 + \frac{y}{z}.$$

(a) Find the vector $\overrightarrow{PQ}$ which points from $P(3, -2, 1)$ to $Q(1, 0, 2)$ as well as the unit vector pointing in the same direction.
(b) Find the rate of change of the potential at the point P in the direction towards the point Q.
(c) In which direction does V change most rapidly at P?
(d) What is the maximum rate of change of V at P?

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Answers:

(a) $PQ = \langle -2, 2, 1\rangle$ and $\mathbf{u} = \langle -\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\rangle$.

(b) $\nabla f = \langle 2x, \frac{1}{z}, -\frac{y}{z^2}\rangle = \langle 6, 1, 2\rangle$, the rate of change is $-\frac{8}{3}$.

(c) in the direction of the gradient, i.e. $\langle 6, 1, 2\rangle$.

(d) $\sqrt{41}$.

(3) Find and classify the critical points of the function

$$f(x,y) = x^2y - 2y^3 + 2x^2 - y^2 + 4$$

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Answer: $(0, -\frac{1}{3})$ is a local minimum, and $(0,0)$, $(2\sqrt{5}, -2)$, $(-2\sqrt{5}, -2)$ are saddle points.

(4) Evaluate the triple integral

$$\iiint_E \frac{z}{\sqrt{x^2+y^2+z^2}}\,dV$$

where E is the lower unit hemisphere:

$$E = \{x^2+y^2+z^2 \le 1, \quad z \le 0\}.$$

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Answer: $-\pi/3$.

(5) Let C be the unit circle $x^2 + y^2 = 1$ oriented counterclockwise. Evaluate

$$\int_C 2y\,dx + x\,dy$$

by two methods: directly and using Green's Theorem.

Answer: $-\pi$.

(6) Compute the area of the surface defined by parametric equations

$$x = u + v, \quad y = u - v, \quad z = 1 - u$$

where $0 \leq u \leq 1$ and $0 \leq v \leq u$.

Answer: $\frac{\sqrt{6}}{2}$.