FALL 2007 — MA 227-6B — TEST 1
SEPTEMBER 19, 2007

Name: _____

1. Part I

There are 6 problems in Part 1, each worth 4 points. Place your answer on the line to the right of the question. Only your answer on the answer line will be graded.

(1) Find the cross product of the vectors $\langle 2, 4, 1\rangle$ and $\langle 0, 1, 1\rangle$.

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(2) Find the dot product of the vectors $\langle 1, -3, 2\rangle$ and $\langle -2, 1, 3\rangle$. _____

(3) Find the derivative of the vector function $\langle \sin(2t), \cos(t), t^2\rangle$.

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(4) Find the indefinite integral $\int (2\mathrm{e}^t\,\mathbf{i} - 6t^2\,\mathbf{k})\,dt$.

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(5) Find a vector function representing the line (a parametrization of the line) passing through the points $P(2, 0, 3)$ and $Q(0, 3, 2)$.

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(6) Find an equation of the plane with normal $\mathbf{i} + 2\,\mathbf{j} - 3\,\mathbf{k}$ which contains the point $P(0, -1, -2)$.

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2. Part II

There are 3 problems in Part 2, each worth 12 points. On Part 2 problems partial credit is awarded where appropriate. Your solution must include enough detail to justify any conclusions you reach in answering the question.

(1) A ball is thrown horizontally from a tower of height $20m$ with an initial speed of $15m/s$.
   (a) Find the vectors of acceleration, velocity, and position.
   (b) How far from the tower will the ball touch ground?
   (c) Find the tangent of the angle at which the ball touches ground?
   (d) Find the ball's speed at impact.
   Use $g = 10m/s^2$.

(2) The curvature of the curve $\mathbf{r}(t)$ at the point $\mathbf{r}(t_0)$ is given by $\kappa(t_0) = |\mathbf{T}'(t_0)|/|\,\mathbf{r}'(t_0)|$ where $\mathbf{T}$ denotes the unit tangent vector.

(a) Compute the curvature for the curve $\mathbf{r}(t) = 2\cos(t)\,\mathbf{i} + 2\sin(t)\,\mathbf{j} + 3t\,\mathbf{k}$ at the point $(-2, 0, 3\pi)$.

(b) Compute the quantity $|\,\mathbf{r}'(\pi) \times \mathbf{r}''(\pi)|/|\,\mathbf{r}'(\pi)|^3$ for the curve given in (a).

Bonus: Show that it is no coincidence that the results in (a) and (b) are the same. Hint: consider $\mathbf{r}'(t) = h(t)\mathbf{T}(t)$ where h describes the magnitude of $\mathbf{r}'$ and $\mathbf{T}$ the direction.

(3) Find an equation of the plane passing through $A(1, 1, -1)$, $B(0, 2, 5)$, and $C(3, 2, 0)$.