MA 227, CALCULUS III

Spring, 2009

Name (Print last name first): .....

Student Signature: .....

TEST 3

10 questions, 10 points each. SHOW ALL YOUR WORK!

Question 1

Find $\int\int_D x\,dxdy$, where D is bounded by $y = 2x^2$ and $y = 4x$.

Question 2

Find the volume under the surface $z = xy$ and above the triangle in the xy plane with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$.

Question 3

Sketch the region of integration and change the order of integration:

$$\int_0^1 \int_{x^2}^1 f(x,y)dydx.$$

Question 4

Use polar coordinates to find the volume under the plane $z = x + 2y + 4$ and above the half-disk $x^2 + y^2 \leq 4,\ y \geq 0$ in xy plane.

Question 5

Find the mass of the lamina that occupies the region:

$$D = \{(x, y)|\ x^2 + y^2 \leq 1,\ x \geq 0\}$$

and has the density function given by $\rho(x, y) = x$.

Question 6

Evaluate the iterated integral $\int_0^1 dy \int_0^{y^2} dx \int_0^y dz$.

Question 7

Express the integral $\int\int\int_E f(x,y,z)dV$ as an iterated integral, where E is the solid above the region $D = \{(x,y) : y^2 \leq x \leq 1\}$ in xy plane and below the plane $z = 1$.

Question 8

Using cylindrical coordinates evaluate $\int\int\int_E (x^2 + y^2)\, dV$, where E is the region that lies inside the cylinder $x^2 + y^2 = 1$ and between planes $z = 0$ and $z = 3$.

Question 9

a) Change $(-1, \sqrt{3}, 0)$ from rectangular to spherical coordinates.

b) Using spherical coordinates evaluate $\int\int\int_E z\, dV$, where E is the ball $x^2 + y^2 + z^2 \leq 1$.

Question 10

Using triple integral in spherical coordinates find the volume of the solid bounded by the surfaces $x^2 + y^2 + z^2 = 1$, $x^2 + y^2 + z^2 = 4$ in the upper half-space $z \geq 0$.