EGR 265, Math Tools for Engineering Problem Solving
September 21, 2011, 50 minutes

Name (Print last name first): ....

 Student ID Number: ....

$$\boxed{\text{TEST I}}$$

Problem 1

Determine the order of the following ODEs. Also, state if they are linear or non-linear. (4P+4P+4P+4P)

(a) $y' + e^x = y$

(b) $y' + e^y = x$

(c) $y^{(4)} - y'' = \cos(x)$

(d) $y^4 - y'' = \cos(x)$

Problem 2

(a) Which of the following functions are solutions of $x^4 y' + 2xy^2 = 12x^5$?(8P)

(i) $y_1 = x^2$

(ii) $y_2 = 2x^2$

(iii) $y_3 = 3x$

(iv) $y_4 = -3x^2$

(b) Which of the functions from part (a) solve the initial value problem $x^4y' + 2xy^2 = 12x^5$, $y(0) = 0$? (4P)

(c)* (Bonus) Does your answer to part (b) agree with the content of the Existence and Uniqueness Theorem for first order ODEs? If yes, why? If no, why not? (5P*)

Problem 3

(a) In the 3×3-grid of points $x = 0, 1, 2$ and $y = 0, 1, 2$ provided in the figure below draw a direction field for $y' = y + x - 3$. (8P)
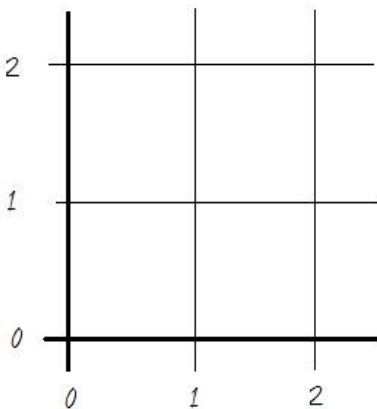
(b) Without solving the DE, use the direction field to guess the solution of the IVP $y' = y + x - 3$, $y(1) = 1$. Check that your guess is correct by verifying that it is a solution of the IVP. (4P)

Problem 4

Solve the IVP (15P)

$$y' = -x^2 y^2, \quad y(1) = 1.$$

Problem 5

Solve the IVP (15P)

$$y' - y = x, \quad y(0) = 1$$

Problem 6

Solve the IVP (15P)

$$y' + \frac{1}{x}y = 2e^{x^2+1}, \quad y(1) = e^2$$

Problem 7

A bag of green beans of 70°F is put to the refrigerator of 20°F. After 10 minutes it is chilled to 50°F.
Note: Your answers to the questions below will contain natural logarithms which do not need to be evaluated.

(a) Newton's Law of Cooling can be used to describe cooling processes as in this problem. Write down the corresponding IVP using an unknown cooling rate k. (4P)

(b) Solve the IVP and determine k by using information provided in the problem. (8P)

(c) When does the temperature of the bean bag reach 30°F? (3P)

SCRATCH PAPER

(Scratch paper will not be graded!)

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