MA 126-5A (Calculus-II), Dr. Chernov — Midterm test #1
Show your work. Each problem is 10 points — Fri, Sep 23, 2005

1. State Fundamental Theorem of Calculus (both parts).

   See the textbook.

2. Find the derivative $f'(x)$ of the function

$$f(x) = \int_{\ln x}^{x^2} \sqrt{1+t^3}\, dt$$

   Answer:

$$f'(x) = 2x\sqrt{1+x^6} - \tfrac{1}{x}\sqrt{1+\ln^2 x}$$

3. Evaluate the indefinite integral $\int \ln(x^5)\,dx$.

Answer:

$$\int \ln(x^5)\,dx = 5\int \ln x\,dx = 5(x\ln x - x) + C$$

4. Evaluate the indefinite integral $\int e^{-2x}\sin x\,dx$.

Answer:

$$\int e^{-2x}\sin x\,dx = \frac{-2e^{-2x}\sin x - e^{-2x}\cos x}{5} + C$$

5. Evaluate the indefinite integral

$$\int \frac{4}{x^2 - 4x}\, dx.$$

Answer:

$$\int \frac{4}{x^2 - 4x}\, dx = \int \frac{1}{x-4} - \frac{1}{x}\, dx = \ln|x-4| - \ln|x| + C$$

6. Evaluate the definite integral $\int_0^{\pi/2} \cos^4 x\, dx$.

Answer:

$$\begin{aligned}\int_0^{\pi/2} \cos^4 x\, dx &= \int_0^{\pi/2} \left(\frac{1+\cos 2x}{2}\right)^2 dx \\ &= \int_0^{\pi/2} \left(\frac{1}{4} + \frac{\cos 2x}{2} + \frac{\cos^2 2x}{4}\right) dx \\ &= \int_0^{\pi/2} \left(\frac{1}{4} + \frac{\cos 2x}{2} + \frac{1+\cos 4x}{8}\right) dx = \frac{3\pi}{16}\end{aligned}$$

7. Determine whether the improper integral

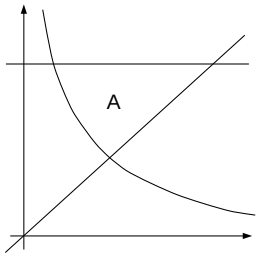
$$\int_0^\infty \frac{x^2}{(x^3+1)^{3/2}}\,dx$$

converges or diverges. If it converges, compute its value.

Answer: using the substitution $u = x^3 + 1$ gives

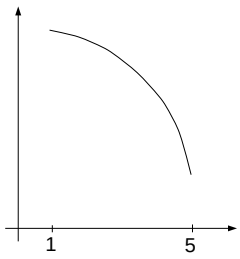
$$\int_0^\infty \frac{x^2}{(x^3+1)^{3/2}}\,dx = \int_1^\infty \frac{1}{3u^{3/2}}\,du = \frac{2}{3}$$

8. Find the area enclosed by the curves $y = x$, $y = 1/x$, and $y = 2$.



Answer: $A = \int_1^2 (y - \frac{1}{y})\,dy = 3 - \ln 2$.

9. Let $I = \int_1^5 f(x)\,dx$, where $f(x)$ is the function whose graph is shown below. For any value of n, list the numbers L_n, R_n, M_n, T_n, and I in increasing order.



Answer: $R_n < T_n < I < M_n < L_n$.

10. Evaluate the definite integral $\int_{-1}^{1} xe^{-x^4}\,dx$ (the answer is simple, but you need to explain it).

Answer: The integral equals zero because the function $f(x) = xe^{-x^4}$ is odd and the interval $(-1, 1)$ is symmetric about zero.

[Bonus] Evaluate the integral

$$\int \frac{\sqrt{x^2+4}}{x^4}\,dx$$

Hint: use the trigonometric substitution $x = 2\tan\theta$.