MA 126-5A (Calculus-II), Dr. Chernov — Midterm test #2
Show your work. — Fri, Oct 28, 2005

1. (10 pts) Find the average value of the function $f(x) = \sqrt{x}$ on the interval $[0, 4]$. Find c such that $f_{\text{ave}} = f(c)$.

Answer: $f_{\text{ave}} = \frac{1}{4}\int_0^4 \sqrt{x}\,dx = 4/3$ and $c = 16/9$.

2. (15 pts) Find the volume of the solid obtained by rotating the region A bounded by the curves $x = 1$, $x = 2$, $y = 0$, $y = 1/x$ about the x axis.

Answer: $V = \pi\int_1^2 x^{-2}\,dx = \pi/2$.

3. (15 pts) Find the radius of convergence and the interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{\sqrt{n}}$$

(do not forget to check the endpoints).

The radius of convergence $R = 1/2$, the interval of convergence $(-1/2, 1/2]$.

4. (10 pts) Express the number $3.24\overline{85}$ as a ratio of integers.

Answer: $\frac{324}{100} + \frac{85}{9900}$.

5. (10 pts) Determine if the following series converges:

$$\sum_{n=0}^{\infty} \frac{2+\cos n}{2^n}$$

Answer: Converges by the comparison test: $2+\cos n \leq 3$, hence the series is dominated by $\sum_{n=0}^{\infty} \frac{3}{2^n}$.

6. (15 pts) Determine if the following series converges:

$$\sum_{n=0}^{\infty} \frac{(-1)^n n^2}{n^3+1}$$

If it does, then does it converge absolutely?

Answer: Converges by the alternative series test. Does not absolutely converge by the limit test (compare it to the harmonic series $\sum \frac{1}{n}$).

7. (15 pts) Determine if the following series converges:

$$\sum_{n=0}^{\infty} \frac{n+1}{\sqrt{n^5+10}}$$

Answer: converges by the limit test (compare it to the p-series $\sum \frac{1}{n^{3/2}}$).

8. (10 pts) Compute the sum of the series $\sum_{n=1}^{\infty} \left[(0.1)^n + (0.5)^{n-1}\right]$.

Answer: $\frac{0.1}{1-0.1} + \frac{1}{1-0.5} = \frac{19}{9}$.

[Bonus] Let $f(x) = kx(2-x)$ if $0 \leq x \leq 2$ and $f(x) = 0$ if $x < 0$ or $x > 2$.
(a) For what values of k is f a probability density function?
(b) For that value of k find $P(X \geq 0.5)$.
(c) Find the mean.

Answers: (a) $k = 3/4$; (b) $81/96$, (c) mean $= 1$.