MA 126-5A (Calculus-II), Dr. Chernov — Midterm test #3
Show your work. — Fri, Dec 2

1. (10 pts) Find the distance from the point $P(3, -1, 0)$ to the plane $2x + 4y - 4z = 14$.

Answer: $d = 2$.

2. (15 pts) Two planes are given: $2x - y - 4z = 5$ and $x = 2z + 1$.
(a) Find parametric equations and symmetric equations for these line of intersection of the planes.
(b) Determine the angle between these planes.

Answers: symmetric equations

$$\frac{x-1}{2} = \frac{y+3}{0} = \frac{z}{1}$$

parametric equations:

$$x = 1 + 2t, \quad y = -3, \quad z = t$$

Angle: $\theta = \cos^{-1} \frac{10}{\sqrt{105}}$.

3. (10 pts) Find the volume of the parallelepiped with adjacent edges PQ, PR, and PS, where $P(2,-1,1)$, $Q(6,0,4)$, $R(3,2,0)$, $S(0,3,5)$.

Answer: $V = 92$.

4. (15 pts) Write its first four nonzero terms of the Maclaurin series for the function $y = \sqrt[3]{1-x^2}$.

Answer:

$$1 - \frac{1}{3}x^2 - \frac{1}{9}x^4 - \frac{5}{81}x^6$$

5. (10 pts) (a) Find a vector perpendicular to the plane through the points $P(1, 0, -2)$, $Q(0, 3, 5)$ and $R(-1, 2, 2)$.
(b) Find the area of the triangle PQR.

Answers: vector is $< -2, -10, 4 >$. Area is $\sqrt{30}$.

6. (15 pts) Find the Taylor series for the function $y = \cos x$ centered at $a = \pi$. Write a general formula for the series and also write down its first three terms.

Answer:

$$-1 + \frac{(x-\pi)^2}{2!} - \frac{(x-\pi)^4}{4!} + \cdots$$

7. (10 pts) Show that the equation

$$2x^2 + 2y^2 + 2z^2 + 12x - 4y + 8z + 10 = 0$$

represents a sphere. Find its center and radius.

Answer: center $(-3, 1, -2)$, radius 3.

8. (15 pts) Recall Maclaurin series for e^x and $\ln(1 + x)$. Then use multiplication to find first three nonzero terms for the Maclaurin series of the function $y = e^{-x} \ln(1 + 2x)$.

Answer:

$$2x - 4x^2 + \frac{17}{3}x^3$$

[Bonus] Find the distance between two skew lines

$$\frac{x+5}{0} = \frac{y-4}{2} = \frac{z+1}{-3}$$

and

$$\frac{x-3}{2} = \frac{y}{6} = \frac{z-2}{0}$$