MA 126-5A (Calculus-II), Dr. Chernov — Final exam
Show your work. — Fri, Dec 9, 2005

1. Find the length of the curve $y = 2\sqrt{x^3}$ between the points $(0, 0)$ and $(1, 2)$.

   Answer: $\frac{2}{27}(10\sqrt{10} - 1)$.

2. Write Maclaurin series for $y = \ln(1-x^2)$ and $y = \cos(4x)$. Then use multiplication find first **three** nonzero terms for the Maclaurin series of the function $y = \ln(1-x^2)\cos(4x)$.

Answer: $-x^2 + \frac{15}{2}x^4 - 7x^6$.

3. Describe the surface given by equation

$$\rho = 2\sin\phi\sin\theta + 4\sin\phi\cos\theta - \cos\phi$$

(First, rewrite this equation in terms of x, y, z.)

Answer: $(x-2)^2 + (y-1)^2 + (z+\frac{1}{2})^2 = \frac{21}{4}$. This is a sphere with center $(2, 1, -\frac{1}{2})$ and radius $\frac{\sqrt{21}}{2}$.

4. Evaluate the indefinite integral $\int e^{-5x} \cos 2x \, dx$. (Work this integral, do not just give an answer.)

Answer: $\frac{2}{29} \sin 2x e^{-5x} - \frac{5}{29} \cos 2x e^{-5x} + C$.

5. (a) Find the equation of the plane through the points $A(2, -1, 1)$, $B(4, 0, -3)$ and $C(0, -2, 0)$.
(b) Find the area of the triangle ABC.

Answer: plane $-5x + 10y = -20$ and area $= \frac{5\sqrt{5}}{2}$.

6. Evaluate the indefinite integral

$$\int \frac{x^2 + x - 4}{x^3 + 4x}\, dx.$$

Answer: $-\ln|x| + \ln(x^2 + 4) + \frac{1}{2}\tan^{-1}\frac{x}{2} + C.$

7. Two planes are given: $x = y + 2z - 2$ and $z = x - 2y + 2$.
(a) Find **parametric equations** and **symmetric equations** for the line of intersection of these planes.
(b) Determine the angle between these planes.

Answers: line $\frac{x+2}{3} = \frac{y}{1} = \frac{z}{1}$; angle $\cos^{-1} \frac{5}{6}$.

8. Determine whether the improper integral

$$\int_1^\infty \frac{e^x}{(e^x-1)^{4/3}}\,dx$$

converges or diverges. If it converges, compute its value.

Answer: it converges; its value is $3/\sqrt[3]{e-1}$.

9. Determine if the following series converges:

$$\sum_{n=0}^{\infty} \frac{(-1)^n n}{n^2 + 5n + 4}$$

If it does, then does it converge absolutely?

Answer: the series converges, but not absolutely.

10. Find first **four** nonzero terms of the Maclaurin series for the function $y = \sqrt[4]{(1-8x)^3}$.

Answer: $1 - 6x - 6x^2 - 20x^3$.

[Bonus] Find the distance between skew lines

$$\frac{x+1}{2} = \frac{y-3}{-1} = \frac{z+1}{0}$$

and

$$\frac{x}{-3} = \frac{y+1}{2} = \frac{z-5}{1}$$