MA 141H-14, Honors calculus - I, Dr Chernov Test I Wed, October 6, 1999 Show your work! Student's name or SSN: ______ Find the following limits: 1. (4 pts)

$$\lim_{x \to -\infty} \frac{4x^3 - 3x^5 + 1}{30 + 1000x^4 - x^5}$$

 $2.~(5~{\rm pts})$

$$\lim_{u \to 0} \frac{\sqrt{u \tan u}}{\sin 2u}$$

3. (4 pts)

$$\lim_{x \to 3} \tan\left(\frac{x^2 - 5x + 6}{x^2 - 2x - 3}\right)$$

4. (10 pts) A function, y = f(x), is graphed below.

At what points is this function discontinuous? Classify the discontinuity points (removable, jumps, infinite, essential...). At what points is the function not differentiable?

Estimate, as best as you can: $\lim_{x \to -2+} f(x) =$ $\lim_{x \to 2} f(x) =$ $\lim_{x \to 3} f(x) =$ f'(0) = f'(-1) = f'(1) = f'(5) =

Sketch the graph of the derivative y' = f'(x).

5. (10 pts) A particle is moving along a horizontal straight line. The graph of its velocity v(t) is shown. When does the particle move to the left? To the right? When is the acceleration positive? Negative? Sketch the graph of the position function f(t) assuming f(0) = 0. Indicate the intervals of upward/downward concavity of f(x). Indicate its inflection points.

6. (7 pts) Use the definition of derivative to differentiate

$$y = \frac{1}{x^2 + x}$$

Find the equation of the tangent line to the graph of this function at the point $(1, \frac{1}{2})$.

For extra credit (8 pts) use the definition of derivative to differentiate

$$y = \frac{1}{x + \sqrt{x}}$$

For extra credit, find the following limits: (6 pts)

$$\lim_{x \to \infty} \sqrt{x^4 + x^2 + x} - \sqrt{x^4 + 1}$$

(6 pts)

$$\lim_{t \to 0} \frac{5\tan 4t}{t^2 \cot 9t}$$

(6 pts)

$$\lim_{x \to \infty} \frac{\sqrt{x} \sin x - x \cos \frac{1}{x}}{x + \sqrt{x} + 1}$$