MA 141-14, Honors Calculus - I   Final Exam   Monday November 22, 1999

Dr Chernov   **Student's name or SSN** ———————————————————

Be sure to show all your work.

1. (5pts) Use Newton's method to approximately solve the equation $x^6 = x^2 + 4$. Find the formula for x_{n+1} in terms of x_n. Take $x_1 = 1$ and compute x_3. Show that there must be a root of this equation on the interval $1 < x < 2$. For extra credit: show that there is exactly one root in the interval $1 < x < 2$.

2. (4 pts) Find the linear approximation of the function $y = \sqrt[n]{x}$ at $a = 1$. Then use it to approximate

$$\frac{\sqrt[4]{1.000000001} - \sqrt[8]{1.000000001}}{0.000000000001}$$

3. (3 pts) A police radar detector shows that a car's speed is 70 mi/h, and 6 seconds later its speed is 60 mi/h. Show that some time during those 6 seconds the car's acceleration was exactly -6000 mi/h^2. State the theorem that you are using.

4. (4 pts) A highway partol plane flies 3 mi above a level, straight road at a steady 120 mi/h. The pilot sees an oncoming car and with radar determines that the line-of-sight distance from plane to car, $s(t)$, is 5 mi and the line-of-sight distance is decreasing at the rate of 160 mi/h. Find the car's speed along the highway.

5. (8 pts) Investigate the function $y = 2\cos x - \sqrt{2}\,x$ on the interval $-\pi \le x \le 3\pi/2$. Find the intervals of increase and decrease, local extrema, the intervals of upward and downward concavity, and inflection points. Find the absolute minimum and absolute maximum of this function. Sketch its graph.

6. (6 pts) Graph the function
$$y = \frac{x^2 + 3}{x - 1}$$
Include all its asymptotes. Find the intervals of increase and decrease, local extrema, the intervals of upward and downward concavity, and inflection points.

For extra credit, do the following problems:

(5 pts) Find the limit

$$\lim_{x\to\infty} \sqrt{x^8 + x^3 \cos x + 1} - x^2\sqrt{x^4 + 2\cos(1/x)}$$

(6 pts) Find the limit

$$\lim_{x\to 0} \frac{\cos 5x - \cos 8x}{e^{2\ln x} \cot 6x \sin 9x}$$

(5 pts) Let a, b, c, d be real numbers. Find the limit

$$\lim_{x \to \infty} \left(\frac{x + a}{x + b} \right)^{cx + d}$$

(4 pts) Show that the equation

$$2x + \sin x + \cos x = 100$$

has exactly one solutions on the interval $(-\infty, \infty)$.

(4 pts) Differentiate the function

$$f(x) = (x + 5)^{x^{\sin x}}$$

(4 pts) With the above function $f(x)$, find the limit $\lim_{x \to 0} f(x)$