Student's name _____

Be sure to show all your work. Every problem is worth 4 points.

1. Solve the initial value problem

$$f'(x) = \sin x - 8x^3 + 3\sqrt{x} - \frac{2}{\sqrt{1 - x^2}} + 4, \qquad f(0) = 1$$

2. Find the limit

$$\lim_{x \to 0} \frac{e^{x^2} - 1}{1 - \cos 4x}$$

3. Compute the integral
$$\int_{-2}^{0} \sqrt{16 - 4x^2}\, dx$$
(Sketch the graph of the function first.)

4. Estimate the following integral
$$\int_{0}^{1} 1 + x\, dx$$

by partitioning the interval $(0, 1)$ into 5 equal subintervals and computing L_5, R_5 and M_5 (i.e. by taking the sample points to be left endpoints, right endpoints and midpoints, respectively). Compute the actual value of the integral. Which estimate is the best?

5. A box with a square base must have a volume of 32 m³. Material for the base costs $5 per square meter, material for the sides costs $3 per quare meter, and material for the top costs $2 per square meter. Find the cost of materials fort the cheapest such container.

6. Find the definite integral

$$\int_{-2}^{2} 6 - 3|x| \, dx$$

7. Solve the boundary value problem

$$f''(t) = 2\sin t - 4\cos t \qquad f(0) = 8 \quad f(\pi) = \pi$$

8. Evaluate the indefinite definite integral

$$\int \frac{1}{\sin^2 x} - \frac{x^2 - 1}{x^4 - 1}\, dx$$

9. Evaluate the definite integral

$$\int_1^2 \frac{1 + (\sqrt{x} - 1)^2}{\sqrt[3]{x^2}} \, dx =$$

10. By differentiating, verify that

$$\int \ln x \, dx = x \ln x - x + C$$

and

$$\int x e^x \, dx = x e^x - e^x + C$$

[Bonus] Evaluate (use only the facts that we discussed in class)

$$\int_{-1}^{1} x\sqrt{x^4 + 1}\, dx$$

[Bonus] Show that for every positive integer $m \geq 1$

$$\sum_{n=1}^{m} \frac{1}{n} > \int_{1}^{n} \frac{1}{x}\, dx$$

Based on this, find the limit

$$\lim_{m \to \infty} \sum_{n=1}^{m} \frac{1}{n}$$