MA 227-6D (Calculus-III), Dr. Chernov — Midterm test #3
Show your work. Each problem is 20 points — Thu, Nov 18, 2004

1. Compute the integral

$$\iint_D (2xy - x^2)\, dA$$

where D is a triangle with vertices $(0, 0)$, $(2, 2)$ and $(2, -2)$.

Answer: -8. The integral setting is

$$\int_0^2 \int_{-x}^{x} (2xy - x^2)\, dy\, dx$$

2. Compute the integral

$$\iiint_E z \, dV$$

where the solid E is bounded by the coordinate planes and the plane $x + y + z = 2$.

Answer: 2/3. The integral setting is

$$\int_0^2 \int_0^{2-x} \int_0^{2-x-y} z \, dz \, dy \, dx$$

3. Compute the integral

$$\iiint_E z\sqrt{x^2+y^2}\,dV$$

where the solid E is inside the cylinder $x^2+y^2=9$ and between the planes $z=1$ and $z=3$.

Answer: 72π. It is better to use cylindrical coordinates:

$$\int_0^{2\pi}\int_0^3\int_1^3 zr^2\,dz\,dr\,d\theta$$

4. Compute the integral

$$\iiint_E \frac{dV}{\sqrt{x^2+y^2}}$$

where the solid E is bounded below by the cone $x^2+y^2=4z^2$ and above by the sphere $x^2+y^2+z^2=36$.

Answer: $36\pi \tan^{-1} 2$. It is better to use spherical coordinates:

$$\int_0^{2\pi} \int_0^{\tan^{-1} 2} \int_0^6 \rho \, d\rho \, d\varphi \, d\theta$$

5. Find the area of the surface with parametric equations

$$\mathbf{r}(u,v) = (2u - v)\mathbf{i} - 2v\mathbf{j} + (u + v)\mathbf{k}, \qquad u^2 + v^2 \leq 4.$$

Answer: $4\pi\sqrt{29}$. The first step: $|\mathbf{r}_u \times \mathbf{r}_v| = \sqrt{29}$.

[Bonus] Two random variables X and Y have joint probability density function

$$f(x, y) = Ce^{-x^2-y^2}$$

where $C > 0$ is a constant. Compute the value of C. Find the expected values μ_X and μ_Y.

Answer: $C = 1/pi$; $\mu_X = \mu_Y = 0$.