MA 263-25 (Numerical Analysis), Dr. Chernov Test II

. Due Thur, Oct 26

1. By running a computer program, find all the real roots (including multiplicities) of the polynomial

$$P(x) = x^5 - x^4 - 3x^3 - 5x^2 - 10x - 6$$

Use Newton's method to find any root, then deflate the polynomial, and so on, until all the real roots are found.

2. You need to compute the integral

$$I = 4 \int_0^1 \frac{dx}{x^2 + 1}$$

to within $d = 10^{-8}$. Find the safe value of n for the midpoint and trapezoid algorithms. Compute the integral by these methods with the safe values of n. Find the actual error (the exact value is $I = \pi$).

3. In problem 2, run both midpoint and trapezoid methods with Richardson's corrections with $n = 4, 6, 8, 10, ...$ until the actual error is $< d = 10^{-8}$. Compare to the results in problem 1. Comments?

4. In problem 2, find the safe value of n for Simpson's algorithm. Then run the Simpson method with that safe value of n.

5. Compute the integral

$$\int_0^1 \sin \sqrt{x}\, dx$$

numerically by the midpoint and trapezoid methods with and without Richardson's modification. Work in double precision. Use values $n = 10, 20, 30, 40, 50$ and make conclusions: which methods converges faster? Comments? For an extra credit, try partitions of variable density to improve the accuracy for small values of n.