MA 485-25 (Probability), Dr. Chernov	Midterm test $\#1$
Show your work. Each problem is 4 pts	Thu, Jan 25 2001

1. An urn contains three white and two red chips. Each time we draw a chip, we look at its color. If it is red, we replace it along with two new red chips. If it is white, we replace it along with one new white chip. What is the probability that, in successive drawing of chips the colors of the first three chips will alternate?

Answer:

$$P(WRW) + P(RWR) = \frac{3}{5} \cdot \frac{2}{6} \cdot \frac{4}{8} + \frac{2}{5} \cdot \frac{3}{7} \cdot \frac{4}{8} = \frac{1}{10} + \frac{3}{35} = 0.1857$$

2. A doctor's office receives 12 calls per hour, on the average. The receptionist takes a 4 minute coffee break.

(a) What is the chance that somebody calls during her coffee break?

(b) What is the chance that at least two people will call?

Answers:  $\lambda = 0.8$ ,

$$P(X \ge 1) = 1 - P(X = 0) = 1 - e^{-0.8} \approx 0.55$$
$$P(X \ge 2) = 1 - P(X = 0) - P(X = 1) = 1 - e^{-0.8} - 0.8e^{-0.8} \approx 0.19$$

3. In a rocket engine, three identical components work in parallel, so that as long as one component works the engine is running. The reliability of the components are 90%, 80%, and 70%, respectively. They work or fail independently from each other.

(a) What is the probability that the engine will run?

(b) Bonus question: find the probability that exactly one component fails.

Answers:

$$P(\text{any component works}) = 1 - P(\text{all fail}) = 1 - 0.1 \cdot 0.2 \cdot 0.3 = 0.994$$

 $P(1 \text{ fails}) = 0.9 \cdot 0.8 \cdot 0.3 + 0.9 \cdot 0.2 \cdot 0.7 + 0.1 \cdot 0.8 \cdot 0.7 = 0.398$ 

4. You roll two dice. Let A = "the dice show different numbers", B = "the first die shows 6", and C = "the second die shows 5 or 6".

(a) Compute the probabilities P(A), P(B), P(C),  $P(A \cap B)$ ,  $P(A \cap C)$ ,  $P(B \cap C)$ , and  $P(A \cap B \cap C)$ .

- (b) Are A and B independent?
- (c) Are A and C independent?
- (d) Are B and C independent?
- (e) Are the three events A, B, and C jointly independent?

Answers: P(A) = 5/6, P(B) = 1/6, P(C) = 1/3;  $P(A \cap B) = 5/36$ ,  $P(A \cap C) = 5/18$ ,  $P(B \cap C) = 1/18$ ,  $P(A \cap B \cap C) = 1/36$ ; Yes, yes, yes, no. 5. A club of N members, is going to elect a committee, which must have a chairman and a secretary (each of which will be a part of the committee). Suppose the size of the committee is  $k \ge 2$ .

(a) How many ways can the committee, including the chairman and the secretary, be chosen?

Answer: 
$$N(N-1)C_{N-2,k-2} = C_{N,k} k(k-1)$$

Now, suppose the size of the committee is not specified, we only know that it must have at least two people – the chairman and the secretary.

(b) How many ways can such a committee be selected?

(c) For an extra credit, you may solve this problem in two different ways, and thus derive a new formula for the binomial coefficients.

Answers:

$$\sum_{k=2}^{N} C_{N,k} k(k-1) = N(N-1)2^{N-2}$$

6. A manufacturer knows that 0.2% of items he produces are defective. He ships items to resellers in boxes of 200 items each. Any box with at least three defective items can be returned for a full refund. What is the probability that a given box will be returned? [Use Poisson approximation!]

Answers:  $\lambda = 0.4$ ,

$$P(X \ge 3) = 1 - P(X = 0) - P(X = 1) - P(X = 2) =$$
$$= 1 - e^{-0.4} - 0.4e^{-0.4} - \frac{(0.4)^2}{2!}e^{-0.4} \approx 0.0079$$

7. A discrete random variable X takes the following values with the corresponding probabilities:

Х	-3	-2	0	1	3	5
Р	0.15	0.1	0.2	0.25	0.1	?

Note that one probability is missing. Assuming that X takes no other values, find the missing probability.

Then Compute the following:

- (a)  $P\{X = 1\} =$  Answer: 0.25
- (b)  $P\{X = -1\} =$  Answer: 0
- (c)  $P\{X \ge 0\}$  = Answer: 0.75
- (d)  $P{X \text{ is positive}} = \text{Answer: } 0.55$
- (e)  $P\{|X| \le 2\} =$  Answer: 0.55
- (f) (conditional probability)  $P\{|X| \le 2/X > 0\} =$  Answer: 5/11
- (g) For an extra credit, describe the probability function of the random variable  $Y = X^2$ .

8. In a college, 82% of the students pass calculus, 80% pass physics, 6% of the students fail both calculus and physics, and 2% pass calculus and physics, but not chemistry. What is the fraction of students that pass all the three courses? [Draw a Venn diagram.]

Answer: 66%

9. Suppose that guilty people do not pass a lie detector test with probability 0.97, and innocent people pass a lie detector test with probability 0.95. During a bank robbery, the police round up six people, among which there are two actual robbers. One suspect is selected at random for a lie detector test.

(a) If the lie detector says that the person is guilty, what is the probability that he really is?

(b) If the lie detector says that the person is innocent, what is the probability that he really is?

Answers: 0.907 to (a) and 0.984 to (b)