MA 485-1E (Probability), Dr. Chernov — Midterm test #1
Show your work. Each problem is 4 pts — Fri, Sep 28, 2001

1. In a rocket engine, three identical components work in parallel, so that as long as one component works the engine is running. The reliability of the components are 80%, 85%, and 90%, respectively. They work or fail independently of each other.

(a) What is the probability that the engine will run?

Answer: $1 - 0.2 \times 0.15 \times 0.1 = 0.997$

(b) What is the probability that exactly two components fail?

Answer: $0.8 \times 0.15 \times 0.1 + 0.2 \times 0.85 \times 0.1 + 0.2 \times 0.15 \times 0.9 = 0.056$

2. A discrete random variable X takes the following values with the corresponding probabilities:

X	2	1	0	-1	-3
P	0.2	0.15	0.25	0.1	?

Note that one probability is missing. Assuming that X takes no other values, find the missing probability.

Then compute the following:

(a) $P\{X \geq 0\}$ = Answer: 0.6

(b) $P\{X = -2\}$ = Answer: 0

(c) $P\{X$ is negative$\}$ = Answer: 0.4

(d) $P\{|X| \leq 1\}$ = Answer: 0.5

(e) (conditional probability) $P\{|X| \leq 1 \,/\, X < 0\}$ = Answer: 0.25

3. An airline company sells 300 tickets for a plane with 298 seats, knowing that the probability a passenger will not show up for the flight is 0.01. Use Poisson approximation to compute the probability they have enough seats for all the passengers who show up.

Solution: Let X be the number of passengers that do not show up for the flight. Then $X = b(300, 0.01)$ and so $X = \text{poisson}(3)$ by Poisson approximation. Now

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1) =$$

$$1 - e^{-3} - 3e^{-3} = 0.801$$

4. In a college, 60% of the students are women and 40% are men. It is known that 30% of the male students use bicycles and 45% of the female students use bicycles.

(a) If a student is selected at random, what is the chance that he/she uses bicycle?

Answer: $P = 0.4 \times 0.3 + 0.6 \times 0.45 = 0.39$

(b) If a student's bicycle is selected at random, what is the chance that it belongs to a woman?

Answer: by Bayes' formula $P = 0.6 \times 0.45/0.39 = 0.69$

5. In a certain city, 50% of the people subscribe to newspaper A, 55% of the people subscribe to newspaper B, 40% of the people subscribe to newspaper C. It is also known that 25% of the people subscribe to A and B, 10% to B and C, and 15% to A and C. No one subscribes to all the three newspapers.

(a) What percentage subscribe to at least one newspaper?

Answer: 95%

(b) What percentage subscribe to exactly one newspaper?

Answer: 45%

(c) Draw a Venn diagram, mark all the relevant parts and their probabilities.