MA 485-1E (Probability), Dr. Chernov — Midterm test #1

Show your work. Each problem is 4 pts — Fri, Sep 27, 2001

1. Diseases D_1, D_2, and D_3 cause symptom A with probabilities 0.5, 0.7, and 0.9, respectively. If 5% of a population have disease D_1, 3% have disease D_2 and 2% have disease D_3, what percent of the population have symptom A? Assume that the only possible causes of symptom A are D_1, D_2, and D_3, and that no one carries more than one of those three diseases.

Answer: by law of total probability 0.025+0.021+0.018 = 0.064

[Bonus] Let a randomly selected person from the population have symptom A. What is the chance he/she carries disease D_2?

Answer: by Bayes rule $0.021/(0.025 + 0.021 + 0.018) = 21/64$

2. A discrete random variable X takes the following values with the corresponding probabilities:

X	-5	-2	0	1	4
P	0.1	0.25	0.15	0.2	?

Note that one probability is missing. Assuming that X takes no other values, find the missing probability.

Answer: 0.3

Then compute the following:

(a) $P\{X \leq 0\} =$ Answer: 0.5

(b) $P\{X \text{ is even}\} =$ Answer: 0.7 (note: 0 is even)

(c) $P\{X^2 \leq 5\} =$ Answer: 0.6

(d) $P\{|X| = 2\} =$ Answer: 0.25

(e) $P\{|X| = 2 \,/\, X < 0\} =$ Answer: $0.25/0.35 = 5/7$

(f) Plot the probability function of X

Note: the plot consists of five vertical line segments, their top points should not be connected.

3. Three missiles are fired at a target and hit it independently, with probabilities 0.8, 0.85 and 0.9, respectively.

(a) What is the probability that the target is hit?

Answer: $1 - 0.2 \times 0.15 \times 0.1 = 0.997$

(b) What is the probability that exactly two missiles hit the target?

Answer: 0.329

4. Suppose only 1% of lottery tickets in a certain state win. Is it enough to buy 100 tickets to ensure that at least one wins? [Just say "yes" or "no".]

Answer: no

In questions (a)–(c) below, use Poisson approximation:
(a) Suppose Jim buys 100 tickets. What is the chance that he wins anything?

Answer: since $\lambda = 1$, we have $P(X \geq 1) = 1 - e^{-1} = 0.6321$

(b) How many tickets one needs to buy to make sure that at least one wins with probability 99%?

Answer: we need $P(X = 0) = e^{-\lambda} \leq 0.01$. Taking logarithm gives $\lambda \geq 4.61$, hence $n \geq 461$. At least 461 tickets are necessary.

(c) Suppose Bob buys 250 tickets. How many of them will win, on the average? What is the chance that at least two tickets win? What is the chance that at most two tickets win?

Answers: $\lambda = 2.5$ (this is average), $P(X \geq 2) = 0.7127$, $P(X \leq 2) = 0.544$

5. An urn contains 5 white balls and 5 black balls. Suppose that 5 persons each draw two balls blindfolded from the urn without replacement. What is the probability that each draws one white ball and one black ball?

Answer:

$$P = \frac{5}{9} \cdot \frac{4}{7} \cdot \frac{3}{5} \cdot \frac{2}{3} \cdot \frac{1}{1} = 0.127$$

Alternatively, one can do it this way:

$$P = \frac{C_{5,1}C_{5,1}}{C_{10,2}} \cdot \frac{C_{4,1}C_{4,1}}{C_{8,2}} \cdot \frac{C_{3,1}C_{3,1}}{C_{6,2}} \cdot \frac{C_{2,1}C_{2,1}}{C_{4,2}} \cdot \frac{C_{1,1}C_{1,1}}{C_{2,2}} = 0.127$$

Same answer.