MA 485-1E (Probability), Dr. Chernov — Midterm test #1

Show your work. Each problem is 20 pts (100 total) — Fri, Sep 26, 2003

1. Two dice are rolled. Let $A = \{$the maximum of the two numbers is 5 or larger$\}$ and $B = \{$the two numbers differ by 1 or less$\}$. Compute the following:

(a) $P(A) =$ Answer: 5/9

(b) $P(B) =$ Answer: 4/9

(c) $P(A \setminus B) =$ Answer: 7/18

(d) $P(A^c \cap B^c) =$ Answer: 1/6

(e) $P(A/B) =$ Answer: 3/8

(f) Are the events A and B independent? Answer: No

The diagram that explains the solution:

	1	2	3	4	5	6
1	B	B			A	A
2	B	B	B		A	A
3		B	B	B	A	A
4			B	B	A,B	A
5	A	A	A	A,B	A,B	A,B
6	A	A	A	A	A,B	A,B

2. A discrete random variable X takes the following values with the corresponding probabilities:

X	-2	-1	0	1	2	3
P	0.1	0.05	?	0.2	0.1	0.15

Note that one probability is missing. Assuming that X takes no other values, find the missing probability.

Answer: 0.4.

Then compute the following:

(a) $P\{X \leq 0\} =$ Answer: 0.55

(b) $P\{1 \leq X^2 \leq 5\} =$ Answer: 0.45

(c) $P\{|X| = 1 \,/\, X > 0\} =$ Answer: 0.2/0.45=4/9

(d) Plot the probability function of X

Partial answer: the plot consists of 6 vertical segments (bars), whose top points are not (!) connected.

[Bonus] Plot the probability function of $Y = X^2$

3. (a) A fair coin is tossed 4 times. What is the probability that the number of heads is an odd number (i.e. not divisible by 2)?

Answer: $(C_{4,1} + C_{4,3})/2^4 = 8/16 = 1/2$.

(b) A fair coin is tossed 200 times. What is the probability that the number of heads is an odd number? (Justify your answer. Just a guess will not count.)

Answer: 1/2. Odd numbers are $1, 3, 5, \ldots, 199$. Even numbers are $0, 2, 4, \ldots, 198, 200$. We want to show that

$$\sum_{k \text{ odd}} C_{200,k} \frac{1}{2^{200}} = \frac{1}{2}$$

Note that in this case, of course,

$$\sum_{k \text{ even}} C_{200,k} \frac{1}{2^{200}} = \frac{1}{2}$$

So we need to check that

$$\sum_{k \text{ odd}} C_{200,k} = \sum_{k \text{ even}} C_{200,k}$$

Moving all the terms to one side, we obtain equation

$$\sum_{k=0}^{200} (-1)^k C_{200,k} = 0$$

This was derived in Remark 1.10 in the classnotes.

4. In a certain city 30% of the people are Conservatives, 50% are Liberals, and 20% are Independents. In a given election, 2/3 of the Conservatives voted, 80% of the Liberals voted, and 50% of the Independents voted.

(a) If we pick a person at random, what is the probability he/she voted?

Answer: 0.7 (by the law of total probability).

(b) If we pick a voter at random, what is the probability he/she is Liberal?

Answer: 4/7 (by the Bayes formula).

5. An insurance company insures 6000 people, each of whom has a 1/2000 chance of an accident in one year. Use the Poisson approximation to find the probability that the number of accidents in one year will be al least 2 and at most 5.

Answer: $\lambda = 6000/2000 = 3$, and

$$P(2 \le X \le 5) = \sum_{k=2}^{5} \frac{\lambda^k}{k!} e^{-\lambda} = 0.716$$