MA 485-1C (Probability), Dr. Chernov	Midterm test $\#1$
6 problems, each is worth 17 points. Show your work.	Mon, Sep 26, 2011

1. You have three alarm clocks that will ring on any given morning with probabilities 0.8, 0.85, and 0.9, respectively. To wake up on an important exam day, you set all the three alarm clocks.

(a) What is the probability of you being awakened by your alarm clocks?

Answer: $1 - 0.2 \times 0.15 \times 0.1 = 0.997$

(b) What is the probability that exactly two alarm clocks will ring?

Answer: $0.2 \times 0.85 \times 0.9 + 0.8 \times 0.15 \times 0.9 + 0.8 \times 0.85 \times 0.1 = 0.329$

2. An insurance company insures 15,000 people, each of whom has a 1/5000 chance of an accident in one year. Use the Poisson approximation to find the probability that the number of accidents in one year will be at least 2 and at most 4. (For full credit, give the answer as a decimal number.)

 $\lambda = 15000 \times (1/5000) = 3$. Probability is

$$\frac{3^2 e^{-3}}{2!} + \frac{3^3 e^{-3}}{3!} + \frac{3^4 e^{-3}}{4!} = \frac{99 e^{-3}}{8} \approx 0.616$$

3. You have 10 dimes and 10 pennies in two boxes. The first box contains 6 dimes and 2 pennies, the second box contains 4 dimes and 8 pennies. You choose a box at random and then pick a coin out of that box at random.

(a) What is the probability that the selected coin is a dime?

Answer: $\frac{3}{4} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} = \frac{13}{24}$

(b) The next day you forget which box you chose but you remember that the selected coin was a dime. What is the chance that you have taken it from the first box?

Answer:

$$\frac{\frac{3}{4} \times \frac{1}{2}}{\frac{3}{4} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2}} = \frac{9}{13}$$

4. A discrete random variable X takes the following values with the corresponding probabilities:

(a) Note that one probability is missing. Assuming that X takes no other values, find the missing probability: $\mathbb{P}(X = 2) = 0.3$

- (b) $\mathbb{P}(0 < X < 3) = 0.55$
- (c) $\mathbb{P}(0 < X < 1) = 0$
- (d) $\mathbb{P}(X \text{ is odd}) = 0.6$
- (e) $\mathbb{P}(|X| = 1) = 0.4$
- (f) conditional probability $\mathbb{P}(X \le 0 / |X| = 1) = 3/8$

[Bonus] Plot the probability function of X

5. In a school lottery, you buy a card with printed numbers $1, 2, \ldots, 10$ and you mark 3 of them. In the end, 3 lucky numbers are selected randomly by the school office, and those who have marked them all will win a prize.

(a) What is the chance that you buy one card and win? (Your answer should be a decimal number or an irreducible fraction.)

Answer: $\frac{1}{C_{10,3}} = \frac{1}{120}$

(b) What is the chance that you correctly guess exactly two out of three lucky numbers?

Answer: $\frac{C_{3,2}C_{7,1}}{C_{10,3}} = \frac{7}{40}$

[Bonus] What is the chance that you buy two cards, mark two *different* triplets and one of your cards wins?

Answer: $\frac{1}{60}$

6. A continuous random variable X has density function

$$f(x) = A\cos x$$

for $-\pi/2 \le x \le \pi/2$ and 0 elsewhere; here A > 0 is some constant.

(a) Find the value of A.

Answer: A = 1/2

(b) Find the distribution function F(x).

Answer: $F(x) = \frac{1+\sin x}{2}$ for $-\pi/2 \le x \le \pi/2$

(c) $\mathbb{P}(-\pi < X < \pi) =$

Answer: 1

(d) $\mathbb{P}(X = \pi/4) =$

Answer: 0

(e)
$$\mathbb{P}(0 < X < \pi/4) =$$

Answer: $\frac{\sqrt{2}}{4}$

(f) $\mathbb{P}(\pi/4 < X < 2\pi) =$

Answer: $\frac{2-\sqrt{2}}{4}$

Recall: $\int \cos x \, dx = \sin x$, $\sin 0 = 0$, $\sin(\pi/2) = 1$, $\sin(-\pi/2) = -1$, $\sin(\pi/4) = \sqrt{2}/2$, and $\sin(\pi) = \sin(-\pi) = 0$.