Midterm test #1 Wed, April 10

1. A random variable X has the following probability distribution (i.e., density):

Compute the following:

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- (a) [1 pt] $P\{X \text{ is positive}\} =$
- (b) [2 pts] $P\{X \le 2 | X > 0\} =$
- (c) [2 pts] EX =
- (d) [2 pts] $EX^2 =$
- (e) [2 pts] VarX =

2. Suppose X and Y are two independent random variables such that EX = 2, VarX = 4, EY = -1 and VarY = 3. Compute the following:

- (a) [2 pts] E(3X + 4Y 3) =
- (b) [3 pts] Var(3X + 4Y 3) =
- (c) [2 pts] $EX^2 =$
- (d) [2 pts] $EY^2 =$
- (e) [3 pts] $E(3X^2 4XY + 2Y^2) =$
- (f) [bonus] Let Z = 3X + 4Y 3. Use Chebyshev's inequality to estimate

 $P\{|Z - EZ| \ge 100\} \le$

(g) [bonus] Let Z = 3X + 4Y - 3. Compute the correlation coefficient of Z and Y. Are Z and Y strongly dependent or weakly dependent?

$$\rho(Z,Y) =$$

3. [10 pts] A manufacturer knows that 0.2% of items he produces are defective. He ships items to resellers in boxes of 125 items each. Any box with three (or more) defective items can be returned for a full refund. What is the probability that a given box will be returned? [Use Poisson approximation!]

4. [7 pts] Statistics show that 1% of men and 0.5% of women smoke. Suppose that 40 men and 20 women are flying on a plane. What is the probability that all the passengers on that plane are nonsmokers? [Use Poisson approximations. Justify the way you combine two Poisson random variables.]

5. A point is selected at random in the unit square. Let

- A be a rectangle bounded by x = 0.1, x = 0.4, y = 0, y = 1,
- B be a rectangle bounded by x = 0, x = 1, y = 0.5, y = 0.8,
- C be a rectangle bounded by x = 0.6, x = 0.9, y = 0, y = 1.

Let D be a triangle bounded by x = 0, y = 1, x = y. See illustrations below. For the following pairs of events determine which are disjoint, independent or neither.

- (a) [1 pt] A and B: disjoint, independent, neither?
- (b) [1 pt] A and C: disjoint, independent, neither?
- (c) [1 pt] B and C: disjoint, independent, neither?
- (d) [1 pt] C and D: disjoint, independent, neither?
- (e) [1 pt] A and D: disjoint, independent, neither?

6. [7 pts] You have 20 dimes and 20 pennies in two boxes. The first box contains 10 dimes and two pennies, the second contains 10 dimes and 18 pennies. You select a box at random and pick a coin in the selected box at random. What is the probability that it is a dime?

7. [bonus] A random variable X takes values $0, 1, 2, \ldots$ Its generating function is

$$\Phi_X(t) = \frac{1}{2-t}$$

Compute the following:

(a) $P\{X = 0\} =$

(b)
$$P\{X=1\} =$$

(c)
$$P\{X=2\} =$$

(d) EX =

(e)
$$\operatorname{Var} X =$$

Can you guess the type of the random variable X?