MA 485-12 (Probability), Chernov — Midterm test #1
Show your work. Each problem is 4 pts. — Wed, April 16

1. A friend flips four coins and tells you that at least one is Tails. Given this, what is the probability that he got 2 Heads and 2 Tails?

Answer:

$P = \frac{C_{4,2}}{2^4 - C_{4,0}} = \frac{6}{15} = 0.4$

2. A club with 40 members is going to form two committees, one with 5 members and the other with 4 members. The committees must have a common chair (who is a part of both committees) but no other common members. How many ways can this be done?

Answer:

we select a chairman, then four more members for the first committee, then three more members for the second:

$40 \cdot C_{39,4} \cdot C_{35,3} = 21,533,311,800$

3. Statistics show that 6% of men and 3% of women smoke. Suppose that 33 men and 66 women are flying on a plane. What is the chance that a passenger on that plane selected at random is a smoker?

Answer:

$0.06 \cdot \frac{33}{99} + 0.03 \cdot \frac{66}{99} = 0.04$

4. A manufacturer knows that 0.2% of items he produces are defective. He ships items to resellers in boxes of 500 items each. Any box with at least three (three or more) defective items can be returned for a full refund. What is the probability that a given box will be returned? [Use Poisson approximation!]

Answer:

$\lambda = 500 \cdot 0.002 = 1$

$$P(n \geq 3) = 1 - p(0) - p(1) - p(2)$$
$$= 1 - e^{-\lambda} - \lambda e^{-\lambda} - \frac{\lambda^2}{2} e^{-\lambda} \approx 0.0803$$

5. You roll a red die and a green die. Let A = "the red die shows a 3", B = "the green die shows a 6", and C = "both dice show the same number". Are A and C disjoint? independent? neither? Answer the same question about A and B. For an extra credit, determine if the three events A, B, and C are independent.

Answer:

$P(A) = 1/6$, $P(B) = 1/6$, $P(C) = 1/6$

$P(A \cap C) = 1/36$, so A and C are independent.

$P(A \cap B) = 1/36$, so A and B are independent.

$P(A \cap B \cap C) = 0$, so A, B, C are not independent.

6. A doctor's office receives 3 calls per hour, on the average. The receptionist takes a 15 minutes coffee break. What is the chance that somebody calls during her coffee break?

Answer:

Average (typical) number of calls per 15 minutes is $3 \cdot 15/60 = 3/4$

This is λ for the Poisson formula.

$P(\geq 1 \text{ calls}) = 1 - P(0 \text{ calls}) = 1 - e^{-\lambda} = 1 - e^{-3/4} \approx 0.5276$

7. Henry is playing in a casino where the chance to win is 0.45. He starts with \$10, makes only \$1 bets, and decides to stop when he doubles his money. What is the probability that he never reaches \$20?

Answer:

$p = 0.45$, $q = 1 - p = 0.55$, $n = 10$, $m = 20$.

$$P(\text{reach } \$20) = \frac{(.55/.45)^{10} - 1}{(.55/.45)^{20} - 1} \approx 0.1185$$

$$P(\text{never reach } \$20) = 1 - 0.1185 = 0.8815$$

8. In an engine, four identical components are connected in parallel, so that as long as one is good the engine is running. The reliabilities of the components are 80%, 75%, 70% and 60%, respectively. What is the probability that the engine will run?

Answer:

$P(\text{engine works}) = 1 - P(\text{engine fails}) = 1 - 0.2 \cdot 0.25 \cdot 0.3 \cdot 0.4 \approx 0.994$

9. [Bonus] A random variable X takes the following values with the corresponding probabilities:

X	-3	-1	2	3
P	0.1	0.3	0.4	0.2

Compute the following:

(a) $P\{X \text{ is positive}\}$ = Answer: 0.6

(b) $P\{|X| \leq 2\}$ = Answer: 0.7

(c) (conditional probability) $P\{X \leq 2 \,|\, X > 0\}$ = Answer: 0.4/0.6=2/3

(d) Draw a diagram of probability distribution for X.