MA 485-12 (Probability), Dr. ChernovMidterm test #1Show your work. Each problem is 4 pts.Wed, Jan 27

1. A friend flips four coins and tells you that at least one is Tails. Given this, what is the probability that he got 1 Heads and 3 Tails?

Answer:

 $P = \frac{C_{4,1}}{2^4 - C_{4,0}} = \frac{4}{15}$

2. A club with 35 members is going to form two committees, one with 6 members and the other with 3 members. The committees must have a common chair (who is a part of both committees) but no other common members. How many ways can this be done?

Answer:

we select a chairman, then five more members for the first committee, then two more members for the second:

 $35 \cdot C_{34,5} \cdot C_{29,2} = 3,954,017,760$

3. Statistics show that 3% of men and 1% of women smoke. Suppose that 40 men and 60 women are flying on a plane. What is the chance that a passenger on that plane selected at random is a smoker?

Answer:

 $0.03 \cdot \frac{40}{100} + 0.01 \cdot \frac{60}{100} = 0.018$

4. A manufacturer knows that 0.1% of items he produces are defective. He ships items to resellers in boxes of 500 items each. Any box with at least three defective items can be returned for a full refund. What is the probability that a given box will be returned? [Use Poisson approximation!]

Answer:

$$\lambda = 500 \cdot 0.001 = 0.5$$

$$P(n \ge 3) = 1 - p(0) - p(1) - p(2)$$

= $1 - e^{-\lambda} - \lambda e^{-\lambda} - \frac{\lambda^2}{2} e^{-\lambda} \approx 0.0144$

5. You roll a red die and a green die. Let A = "the red die shows a 2", B = "the green die shows a 5", and C = "both dice show the same number". Are A and C disjoint? independent? neither? Answer the same question about A and B. Determine whether the three events A, B, and C are (jointly) independent.

Answer:

P(A) = 1/6, P(B) = 1/6, P(C) = 1/6

 $P(A \cap C) = 1/36$, so A and C are independent.

 $P(A \cap B) = 1/36$, so A and B are independent.

 $P(A \cap B \cap C) = 0$, so A, B, C are not independent.

6. In an engine, four identical components work in parallel, so that as long as one is in good shape the engine is running. The reliabilities of the components are 80%, 75%, 65% and 60%, respectively. They fail independently. What is the probability that the engine will run? Bonus question: find the probability that exactly one component fails.

Answer:

 $P(\text{engine works}) = 1 - P(\text{engine fails}) = 1 - 0.2 \cdot 0.25 \cdot 0.35 \cdot 0.4 \approx 0.993$

Compute the following:

(a) $P\{X = 2\}$ = Answer: 0.25

- (b) $P\{X = -2\} =$ Answer: 0
- (c) $P\{X \le 1\} =$ Answer: 0.6
- (d) $P{X \text{ is positive}} = \text{Answer: } 0.6$
- (e) $P\{|X| \le 2\} =$ Answer: 0.75
- (f) (conditional probability) $P\{X \leq 2 \,|\, X > 0\} =$ Answer: 0.45/0.6=0.75
- (d) [Bonus] Sketch the diagram of the density function of X.

8. A random variable X has the following density function:

$$f(x) = \begin{cases} 3x^2/2 & \text{for } 0 < x \le 1\\ 1/2 & \text{for } 1 < x \le 2 \end{cases}$$

(a) Find the distribution function of X

$$F(x) = \begin{cases} \int_0^x 3x^2/2 \, dx = \frac{x^3}{2} & \text{for } 0 \le x \le 1\\ \int_0^x f(x) \, dx = \frac{1}{2} + \frac{x-1}{2} & \text{for } 1 \le x \le 2 \end{cases}$$

(b) $P(1/2 < X < 4) = \text{Answer: } 1 - \frac{1}{8} = \frac{7}{8}$

(c) P(X = 1) = Answer: 0

(d)
$$P(0.9 < X < 1.5) =$$
 Answer: $\frac{1.5}{2} - \frac{(0.9)^3}{2} = 0.3855$

- (e) F(4) = Answer: 1
- (f) F(-2) = Answer: 0
- (g) f(4) = Answer: 0
- (h) Sketch the graph of the distribution function.