

1. A continuous random variable X has distribution function

$$F(x) = 1 - x^2$$

for $-1 \leq x \leq 0$. Compute the following:

- (a) the density function

$$f(x) = \quad .$$

Answer: $f(x) = -2x$ for $-1 \leq x \leq 0$.

- (b) $E(X)$ = Answer: $-2/3$.

- (c) $E(X^2)$ = Answer: $1/2$.

- (d) $\text{Var}(X)$ = Answer: $1/18$.

- (e) σ_X = Answer: $1/\sqrt{18}$.

(Bonus) Find the moments of X of all orders $k \geq 1$.

Answer:

$$E(X^k) = \frac{2(-1)^{k+2}}{k+2}$$

2. Let X be an exponential random variable with half-life $t_{1/2} = 3$.

(a) Find the parameter λ . Answer: $\lambda = (\ln 2)/3$

(b) Compute $P(X > 9)$. Answer: $1/8$.

(c) Find the conditional probability $P(X > 101/X > 98)$ Answer: $1/2$.

3. A continuous random variable X has density function $f(x) = cx - 2$ for $1 < x \leq 2$ (and zero elsewhere).

(a) Find the the value of c . Answer: $c = 2$.

(b) Find the distribution function $F(x)$ of X . Answer: $F(x) = (x - 1)^2$ for $1 < x \leq 2$.

(c) Find $P(1.6 < X < 16)$. Answer: 0.64.

(d) Find $P(X = 1)$. Answer: 0.

4. By using the table for $\Phi(x)$ on page 488, find the following probabilities for two normal random variables, $Z = N(0, 1)$ and $X = N(3, 16)$.

(a) $P(-1.45 < Z < -0.67) =$ Answer: 0.1779.

(b) $P(X > 3.88) =$ Answer: 0.4129.

(c) $P(2.4 < X \leq 24) =$ Answer: 0.5596.

(d) What is the type (and parameters) of the random variable $Y = 2(4 - X)$? Answer: $Y = N(2, 64)$.

5. Let X be a uniform random variable $X = U(0, 1)$.

Find the distribution and density functions for the random variable $Y = \frac{6}{X-2}$.

Range: $-6 < Y < -3$. The distribution function:

$$\begin{aligned} F_Y(y) &= P(Y \leq y) = P\left(\frac{6}{X-2} \leq y\right) = P((X-2)y \leq 6) \\ &= P\left(X-2 \geq \frac{6}{y}\right) = P\left(X \geq \frac{6}{y} + 2\right) = 1 - F_X\left(\frac{6}{y} + 2\right) = 1 - \left(\frac{6}{y} + 2\right) = -1 - \frac{6}{y} \end{aligned}$$

The density function:

$$f(y) = \frac{6}{y^2}$$

[Bonus] Find $E(Y)$

6. Random variables X and Y are independent and uniformly distributed on the interval $(1, 3)$, i.e. $X = U(1, 3)$ and $Y = U(1, 3)$.

(a) Find the joint density function $f_{X,Y}(x, y)$. Answer: $f_{X,Y}(x, y) = 1/4$ for $1 \leq x, y \leq 3$ and zero elsewhere.

(b) Find the probability $P(|X - Y| < 1)$. Answer: $3/4$.

(c) Find the probability $P(X + Y = 4)$. Answer: 0 .