$\begin{array}{c} \mbox{Midterm test } \#2 \\ \mbox{Wed, May 1} \end{array}$

Student name____

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1. A random variable X has the following distribution function:

$$F(x) = \begin{cases} \frac{(x+1)^2}{8} & \text{for } -1 < x \le 1\\ \frac{x}{2} & \text{for } 1 < x \le 2 \end{cases}$$

- (a) (1 pt) Find F(-1), F(1) and F(2). Is this random variable continuous? Why?
- (b) (1 pt) Find the density of X

$$f(x) =$$

- (c) (1 pt) P(X > 0) =
- (d) (1 pt) P(X = 1) =
- (e) (1 pt) P(X < 1|X > 0) =

2. The lifetime of a VCR is an exponential random variable X with half-lihe $\bar{t}_{1/2} = 3$ (years).

- (a) (1 pt) Find the parameter $\lambda =$
- (b) (1 pt) Write down the distribution function F(x) =

and the density function f(x) =

- (d) (1 pt) Write down the mean life time EX =
- (e) (1 pt) Write down the variance VarX =
- (f) (1 pt) P(X > 9) =
- (g) (1 pt) Compute the median m =
- (e) (2 pts) Compute both quartiles

$$q_{1/4} =$$

 $q_{3/4} =$

3. (6 pts) Candidates A and B are running for office and 70% of the electorate favor candidate A. What is the proability that in a poll where 400 people are questioned at least 145 support candidate B? (Use normal approximation.)

Bonus question: Replace 145 by 150 and solve the problem by using the 'three sigma rule'.

4. (5 pts) A discrete random variable X takes the following values with the corresponding probabilities:

Sketch the graph of its distribution function $F_X(x)$. Indicate clearly which endpoints of segments are included and which are not.

5. A random variable X has the density $f(x) = \frac{3}{2}x^2$ for $-1 \le x \le 1$. (a) (1 pt) P(|X| > 0.5) =

- (b) (1 pt) EX =
- (c) (1 pt) $EX^2 =$
- (d) (1 pt) VarX =
- (e) (1 pt) $\sigma_X =$
- (f) [Bonus] Find the median and the mode of X.
- (g) [Bonus] Find the moments M_r of all orders $r \ge 1$.

6. A fair coin is tossed until 200 heads appear. By using normal approximation,

(a) (5 pts) find the probability that exactly 410 tosses will be necessary (what theorem do you use?);

(b) (3 pts) find the probability that at least 410 tosses will be necessary (what theorem do you use?).

7. (6 pts) Suppose that X is a uniform random variable on the unit interval, [0, 1]. Find the distribution function F_Y and the density function f_Y for $Y = 1 - \sqrt{X}$. Sketch the graph of f_Y .

Bonus question: find EY.

8. (7 pts) Suppose that the lifetime of computer diskettes used in a computer store for demonstration is exponentially distributed with a mean lifetime 1 month. As soon as one diskette breaks, a new one is installed in its place. Find the probability that more than 25 diskettes will be required during a two-year period. (Use normal approximation, name the theorem that you use.)