MA 485-12, Probability (Dr Chernov) Final Exam Wed, June 4, 1997

Student's name ____________________

Be sure to show all your work. Every problem is worth 4 points.

1. A discrete random variable X takes the following values, with the corresponding probabilities:

X	-3	-1	2	3
P	0.1	0.3	0.4	0.2

Compute the following:

(a) $EX =$

(b) $EX^2 =$

(c) $EX^3 =$

(d) $\mathrm{Var}X =$

(e) $\sigma(X) =$

(f) Bonus: $\phi_X(t) =$

2. A continuous random variable X has the distribution function $F(x) = x^3/8$ for $0 \leq x \leq 2$. Comute the following:

(a) $F(1) =$ $\qquad$ $F(3) =$ $\qquad$ $F(-1) =$

(b) the density function $f(x) =$

(c) $EX =$

(d) $EX^2 =$

(e) $\mathrm{Var}X =$

(f) $\sigma(X) =$

(g) Bonus: $\phi_X(t) =$

3. A random variable X has generating function

$$\gamma_X(z) = \frac{1}{2-z}$$

Compute the following:

(a) $EX =$

(b) $E\{X(X-1)\} =$ [Remember, $E\{X(X-1)\} = \gamma_X''(1)$]

(c) $EX^2 =$

(d) $\text{Var}X =$

(e) $\sigma(X) =$

(f) Bonus: Compute $P(X=0)$, $P(X=1)$, etc., and guess what type of probability distribution this is.

4. For two random variables, X and Y, it is known that $EX = -4$, $\sigma(X) = 5$, $EY = 3$, and $\sigma(Y) = 2$. Let $Z = 5 - 2Y + 4X$.

(a) Compute $EZ =$

(b) Assume that X and Y are independent and compute $\mathrm{Var}Z$ and $\sigma(Z)$.

(c) Bonus: assume that $\rho(X, Y) = -0.2$ and compute $\mathrm{Var}Z$.

5. Let X be a standard normal random variable, and $Y = X^3$.

(a) Recall EX and $\text{Var}X$

(b) Find EY and $\text{Var}Y$ (recall that $EX^6 = 15$).

(c) Find $\text{cov}(X, Y)$ and $\rho(X, Y)$. Are X and Y independent, weakly dependent, or strongly dependent?

(d) Based on Chebyshev's inequality, estimate $P(|Y| \geq 10)$.

(e) Bonus: Find the exact value of $P(|Y| \geq 10)$.

6. An operator is taking phone calls, at a rate of 0.8 calls per minute. Let X be the number of calls received during a day ($= 24$ hours).

(a) What is the type of the random variable X? (Include the parameter value.)

(b) By using normal approximation, estimate $P(X > 1200)$.

(c) By using the 3σ-rule, predict the minimum and maximum number of calls over a 24 hours period.

(d) Bonus: Let $X = \text{Poisson}(\lambda)$ and $Y = \sqrt{X}$. Estimate $\sigma(Y)$ for large λ (i.e., when $\lambda \to \infty$).

7. The lifetime of a light bulb is an exponential random variable with half-life $\bar{t}_{1/2} = 8.32$ (hours). (Note that $8.32 \approx 12 \ln 2$.) You have a 25-pack of light bulbs. Let X be their total lifetime (the lifetime of all the 25 bulbs, combined).

(a) What is the mean value and varianve of X?

(b) Use the central limit theorem to estimate $P(X < 220)$. Do you have to use histogram correction here?

8. A student knows about 25% of answers in an oral test. The professor keeps asking questions until the student answers 5 of them correctly. What is the probability that more than 25 questions will be necessary?

9. Let $X_1, \ldots, X_{400}$ be i.i.d. random variables, uniformly distributed on $(0, 1)$. Let $\bar{X} = (X_1 + \cdots + X_{400})/400$. Estimate $P(\bar{X} > 0.48)$.

10. A random variable X has distribution function $F(x) = x^2$ for $0 < x < 1$. Compute the distribution function $G(x)$ and the density function $g(x)$ of the random variable Y, where

(a) $Y = 2 - \frac{1}{X}$.

(b) $Y = X^2 + 2$.

(c) Bonus: $Y = \ln X$.