MA 486-1E (Statistics), Chernov — Midterm test #1

Show your work. Each problem is worth 5 points. — Mon, Feb 10, 2003

1. The following are quiz scores in a calculus class:

3	12	19	1	6	17	5	20	14	10
8	0	15	11	2	18	1	13	9	16

(a) Determine the mode, the median, the quartiles and the IQR.

(b) Find the sample mean, the sample variance and the sample standard deviation.

Answers:

mode=1, median=10.5, the first quartile is 4, the third quartile is 15.5, IQR=11.5.

Next, $\bar{x} = 10$, $s^2 \approx 42.42$, $s \approx 6.5$.

2. Let p denote the proportion of defective items manufactured in a production line. A quality test of randomly selected 120 items shows that 21 of them were defective. Give the lower endpoint of a one-sided 99% confidence interval for p.

Answer: Since $n = 120$ is larger than 30, we can use normal approximation. Hence

$$p > 1.175 - 2.236\sqrt{\frac{0.175 \times 0.825}{120}} = 0.094$$

where we used $z_{0.01} = 2.236$.

3. A random sample of size $n = 23$ from $N(\mu, \sigma^2)$ yielded $\bar{x} = 4.0$ and $s^2 = 1.96$. Construct a 90% confidence interval for σ.

[Bonus] Construct the shortest 90% confidence interval for σ.

Answer:

$$\frac{22 \times 1.96}{33.92} < \sigma^2 < \frac{22 \times 1.96}{12.34}$$

or

$$1.1275 < \sigma < 1.8693$$

Bonus:

$$\frac{22 \times 1.96}{36.646} < \sigma^2 < \frac{22 \times 1.96}{13.253}$$

or

$$1.0847 < \sigma^2 < 1.8038$$

4. A random sample $x_1, \ldots, x_{16}$ from $N(\mu, 64)$ yielded

$$\sum_{i=1}^{16} x_i = 56 \qquad \text{and} \qquad \sum_{i=1}^{16} x_i^2 = 198$$

(a) Give a point estimate for μ.
(b) Construct a 98% confidence interval for μ.

Answer: Note that $\sigma^2 = 64$ is known, so we do not need to compute s^2. The point estimate is $\hat{\mu} = 3.5$. The confidence interval is

$$3.5 \pm 2.326 \times 8/4$$

which is $[-1.152, 8.152]$. We used $z_{0.01} = 2.236$.

5. A committee wants to estimate the proportion p of employees in a large company that support a certain innovation. How many employees should be polled to estimate p to within 0.05 at a 95% confidence level?

Answer:
Since p is completely unknown, we use its value $p^* = 0.5$ and get

$$n \geq \frac{(1.960)^2}{4 \times 0.05^2} = 384.16$$

hence $n \geq 385$.

6. The math test scores in two classes produced the following results: $n_x = 20$, $\bar{x} = 66$ and $s_x = 5.3$ for the first class and $n_y = 10$, $\bar{y} = 59$ and $s_y = 5.3$ for the second class. Assume that $\sigma_x = \sigma_y$ and construct a 90% confidence interval for the difference $\mu_x - \mu_y$.

Answer: we use the formula for equal sigmas. The CI is

$$7 \pm 1.701 \times \sqrt{\frac{19 \times 28.09 + 9 \times 28.09}{28}} \times \sqrt{\frac{1}{20} + \frac{1}{10}}$$

which is $[3.5084, 10.4916]$. Here $t_{0.05}(28) = 1.701$.

7. Let $x_1, \ldots, x_{16}$ and $y_1, \ldots, y_9$ be two independent random samples from distributions $N(\mu_x, \sigma_x^2)$ and $N(\mu_y, \sigma_y^2)$, respectively. Their sample means are $\bar{x} = 3.2$ and $\bar{y} = -1.0$. Their sample variances are $s_x^2 = 4.1$ and $s_y^2 = 3.9$, respectively. Construct a 98% confidence interval for ratio of standard deviations σ_x/σ_y.

Answer:

$$\frac{4.1/3.9}{5.52} < \frac{\sigma_x^2}{\sigma_y^2} < \frac{4.1}{3.9} \times 4.00$$

hence

$$0.44 < \sigma_x/\sigma_y < 2.05$$

Here $F_{0.01}(15, 8) = 5.52$ and $F_{0.05}(8, 15) = 4.00$.

8. Let $x_1, \ldots, x_n$ be a random sample from the distribution with probability density function

$$f(x;\theta) = \theta(1-x)^\theta \quad \text{for } \ 0 < x < 1$$

where $\theta > 0$ is an unknown parameter. Find the maximum likelihood estimate for θ.

[Bonus] Find the asymptotic mean value and variance of the maximum likelihood estimate of θ by using the material of Section 9.6, pages 590–591.

Answer:

$$L(\theta) = \theta^n(1-x_1)^\theta \cdots (1-x_n)^\theta$$

$$\ln L(\theta) = n\ln\theta + \theta\sum_{i=1}^n \ln(1-x_i)$$

$$\frac{d\ \ln L(\theta)}{d\theta} = \frac{n}{\theta} + \sum_{i=1}^n \ln(1-x_i)$$

$$\hat{\theta} = -\frac{n}{\sum_{i=1}^n \ln(1-x_i)}$$

Let [Bonus] Let $x_1, \dots, x_n$ be a random sample from the normal distribution $N(\theta, \theta)$, whose mean value and variance are equal. Find sufficient statistics.

Answer: the only sufficient statistic is $\sum_{i=1}^n x_i^2$.