

MA 486-1E (Statistics), Dr Chernov
Show your work.

Midterm test #1
Fri, Feb 6, 2004

1. (12 pts) Let p_1 be the proportion of adult men that smoke and p_2 the proportion of adult women that smoke. In a random sample of 300 men, 36 are smokers. In a random sample of 400 women, 32 are smokers. Construct a 98% confidence interval for $p_1 - p_2$.

Answer: $[-0.014, 0.094]$.

2. (12 pts) A random sample of size $n = 15$ from $N(\mu, \sigma^2)$ yielded

$$\sum_{i=1}^{15} x_i = -15 \quad \text{and} \quad \sum_{i=1}^{15} x_i^2 = 43$$

(a) Compute a maximum likelihood estimate for σ^2 .

Answer: $28/15$.

(b) Compute an unbiased estimate for σ^2 .

Answer: 2.

(c) Construct a 95% confidence interval for σ^2 .

Answer: $[1.07, 4.97]$.

[Bonus] Give the lower endpoint of a one-sided 95% confidence interval for σ^2 .

Answer: 1.18.

3. (14 pts) Let x_1, \dots, x_n be a random sample from the distribution with probability density function

$$f(x; \theta) = (2\theta + 1)x^{2\theta} \quad \text{for } 0 < x < 1$$

where θ is an unknown parameter.

(a) Find the maximum likelihood estimate for θ .

Answer:

$$\hat{\theta} = \frac{n - \sum \ln x_i}{2 \sum \ln x_i}$$

(b) Find sufficient statistics, and make sure that the MLE is a function of them.

Answer: $\prod x_i$.

[Bonus] Find the Rao-Cramer lower bound on the variance of the unbiased estimates of θ .

Answer:

$$\text{Var } \hat{\theta} \geq \frac{(2\theta + 1)^2}{4n}$$

4. (12 pts) Let p denote the proportion of defective items manufactured in a production line. A quality test of randomly selected 60 items shows that 9 of them were defective. Give the upper endpoint of a one-sided 95% confidence interval for p .

Answer: 0.2258

5. (12 pts) Let x_1, \dots, x_{21} and y_1, \dots, y_{61} be two independent random samples from distributions $N(\mu_x, \sigma_x^2)$ and $N(\mu_y, \sigma_y^2)$, respectively. Their sample means are $\bar{x} = -1.3$ and $\bar{y} = 5.7$. Their sample standard deviations are $s_x = 3$ and $s_y = 6$, respectively. Construct a 90% confidence interval for ratio of standard deviations σ_x/σ_y .

Answer: $[0.378, 0.698]$.

6. (14 pts) The math test scores in two classes produced the following results: $n_x = 18$, $\bar{x} = 75$ and $s_x = 6$ for the first class and $n_y = 16$, $\bar{y} = 88$ and $s_y = 4$ for the second class. Construct an 80% confidence interval for the difference $\mu_x - \mu_y$. [Warning: make sure that you are using the right formula! Check the assumptions made in this problem.]

Hint: Use Welch's formula. The correct number of degrees of freedom should be $r = 29$.

7. (12 pts) The following are quiz scores in a calculus class:

9 13 6 4 0 11 15 3 6 9 4 4

(a) Determine the mode, the median, the quartiles and the IQR.

Answers: 4, 6, 4 and 10, 6.

(b) Find the sample mean, the sample variance and the sample standard deviation.

Answers: 7, 19.82, 4.45.

8. (12 pts) A random sample x_1, \dots, x_{23} from $N(\mu, \sigma^2)$ yielded

$$\sum_{i=1}^{23} x_i = 69 \quad \text{and} \quad \sum_{i=1}^{23} x_i^2 = 295$$

(a) Give a point estimate for μ .

Answer: 3.

(b) Construct a 99% confidence interval for μ .

Answer: 3 ± 1.1756 .

(c) Construct the shortest 99% confidence interval for μ .

Answer: the same as in (b).