MA 486/586-1G (Statistics), Dr Chernov — Midterm test #1

Show your work. — Fri, Feb 9, 2007

1. (20 pts) The following are changes in the price of a stock over 10 consecutive days:

$8 \quad 22 \quad -4 \quad 9 \quad -1 \quad 14 \quad 11 \quad 2 \quad 0 \quad -1$

(a) Determine the mode, the median, the quartiles and the IQR.

Answers: mode$= -1$, median$= 5$, 1st quartile$= -1$, 3rd quartile$= 11$ (some other methods yield the value of 11.75, which is acceptable); IQR$= 12$ (respectively, 12.75).

(b) Draw a Box-and-Whisker diagram.

(c) Find the sample mean, the sample variance and the sample standard deviation.

Answers: $\bar{x} = 6$, $s^2 \approx 67.5$, $s \approx 8.2$.

2. (30 pts) A random sample of size $n = 81$ from $N(\mu, \sigma^2)$ yielded

$$\sum_{i=1}^{81} x_i = 162 \qquad \text{and} \qquad \sum_{i=1}^{81} x_i^2 = 1044$$

(a) Compute a maximum likelihood estimate for μ. Did you use an unbiased estimate?

Answer: $\hat{\mu} = \bar{x} = 2$.

(b) Construct a 96% confidence interval for μ.

Answer: $2 \pm 2.054 \times 3/9$ or $[1.3, 2.7]$.

(c) Compute a maximum likelihood estimate for σ^2.

Answer: $V = 720/81 \approx 8.9$.

(d) Compute an unbiased estimate for σ^2.

Answer: $s^2 = 720/80 = 9$.

(e) Construct a 95% confidence interval for σ^2 using the χ^2 percentiles.

Answer:

$$\frac{80 \times 9}{106.6} < \sigma^2 < \frac{80 \times 9}{57.15}$$

or $[6.75, 12.6]$.

(Bonus) Construct a 95% confidence interval for σ^2 using normal approximation.

Answer:

$$\frac{9\sqrt{80}}{\sqrt{80} + 1.96\sqrt{2}} < \sigma^2 < \frac{9\sqrt{80}}{\sqrt{80} - 1.96\sqrt{2}}$$

or $[6.87, 13.0]$.

3. (30 pts) Let $x_1, \ldots, x_{16}$ and $y_1, \ldots, y_7$ be two independent random samples from normal distributions $N(\mu_x, \sigma_x^2)$ and $N(\mu_y, \sigma_y^2)$, respectively. Their sample means are $\bar{x} = 3.1$ and $\bar{y} = -0.4$, and their sample variances are $s_x^2 = 24$ and $s_y^2 = 36$, respectively.
(a) Construct a 90% confidence interval for the ratio σ_x/σ_y.

Answer:

$$\frac{24}{36} \times \frac{1}{3.94} < \frac{\sigma_x^2}{\sigma_y^2} < \frac{24}{36} \times 2.79$$

or $[0.411, 1.36]$.

(b) Assume that $\sigma_x = \sigma_y$ and give an upper endpoint for a one-sided 90% confidence interval for the difference $\mu_x - \mu_y$.

Answer:

$$3.1 + 0.4 + 1.323 \sqrt{\frac{15 \times 24 + 6 \times 36}{21}} \sqrt{\frac{1}{16} + \frac{1}{7}} = 6.62$$

4. (20 pts) Suppose in a sequence of 100 independent trials 30 successes (and 70 failures) are observed. Let p denote the unknown probability of success.
(a) Give an unbiased estimate $\hat{p}$ for p.

Answer: $\hat{p} = 0.3$.

(b) Find (approximately) the standard deviation of your estimate;

Answer: $\sqrt{0.3 \times 0.7/100} \approx 0.046$.

(c) Construct a 97.5% confidence interval for p.

Answer:

$$0.3 \pm 2.24\sqrt{\frac{0.3 \times 0.7}{100}}$$

or $[0.197, 0.403]$.

(Bonus) Use the exact formulas (section 9.2 of the classnotes) to construct a 97.5% confidence interval for p.

Answer:

$$\frac{0.3 + 2.24^2/200}{1 + 2.24^2/100} \pm \frac{2.24\sqrt{\frac{0.3\times 0.7}{100} + \frac{2.24^2}{4\times 100^2}}}{1 + \frac{2.24^2}{100}}$$

or $[0.209, 0.409]$.