MA 486-12 (Statistics), Chernov — Midterm test #1
Show your work. Each problem is 4 pts. — Wed, April 14

1. The following are quiz scores in a calculus class:

6	4	10	8	2	7	7	9	4	10
1	5	7	9	4	9	3	6	5	4

(a) Construct a frequency table for these data.
(b) Draw a histogram.
(c) Determine the mode, the median, the quartiles and the IQR.
(d) Draw a Box-and-Whisker diagram.
(e) Find the sample mean, the sample variance and the sample standard deviation.

Some answers:

mode = 4, median = 6, quartiles = 4 and 8.5, IRQ = 4.5

sample mean = 6.0, sample variance = 7.053, sample st. deviation = 2.656

2. Let $x_1, \ldots, x_n$ be a random sample from the distribution with probability density function

$$f(x;\theta) = \frac{1}{2\theta^3} x^2 e^{-x/\theta} \quad \text{for} \quad x > 0$$

Find the maximum likelihood estimator for θ. Is this estimator unbiased? (Hint: $\int_0^\infty x^3 e^{-ax}\, dx = 6a^{-4}$ for $a > 0$.)

A partial answer: the MLE is $\hat{\theta} = \bar{x}/3$

3. [Bonus] In problem 2, determine the sufficient statistics. Also, find the Rao-Cramer lower bound of the maximum likelihood estimator.

Sufficient statistic is $\sum x_i$.

4. During the Friday night shift, $n = 81$ mints were selected at random from a production line and weighed. They had an average weight of $\bar{x} = 19.67$ grams and $s = 0.26$ grams.

(a) Construct a 95% confidence interval for μ, the mean weight of all the mints.
(b) Give the lower endpoint of a 90% one-sided confidence interval for μ.

Answers:

(a) the CI is $[19.61, 19.73]$

(b) the CI is $[19.63, +\infty)$

5. The SAT test scores in two schools produced the following results: $n_x = 12$, $\bar{x} = 480$ and $s_x = 50$ for the first school and $n_y = 16$, $\bar{y} = 560$ and $s_y = 60$ for the second school. Construct a 90% confidence interval for the difference $\mu_x - \mu_y$. [Note: since σ_x and σ_y are unknown and may be different, use Welsh's T.]

The Welsh's T method is not covered in 2000.

6. A random sample of $n = 15$ wheels of cheese yielded the sample mean $\bar{x} = 19.87$ pounds and the sample standard deviation $s = 1.60$ of their weights. Assume that the weight of a wheel of cheese is a normal random variable $N(\mu, \sigma^2)$.

(a) Find a 90% confidence interval for σ that cuts off 5% probability on the left side and 5% on the right side.
(b) Also, find the shortest 90% confidence interval for σ.

Answers:

(a) The CI is $[1.23, 2.34]$

(b) The CI is $[1.16, 2.21]$

7. Let $x_1, \ldots, x_{16}$ and $y_1, \ldots, y_{21}$ be two independent random samples from the distributions $N(\mu_x, \sigma_x^2)$ and $N(\mu_y, \sigma_y^2)$, respectively. Their sample mean values are $\bar{x} = 8.13$ and $\bar{y} = 12.67$. Their sample variances are $s_x^2 = 4.56$ and $s_y^2 = 3.94$, respectively.

(a) Give a point estimate for σ_x^2/σ_y^2.
(b) Find a 98% confidence interval for σ_x^2/σ_y^2.
(c) Find a 95% confidence interval for σ_x^2/σ_y^2.

Answers:

(a) point estimate is 1.157

(b) The CI is $[0.374, 3.899]$

(c) The CI is $[0.450, 3.193]$

8. Let p equal the proportion of Americans who select jogging as one of their recreational activities. A poll shows that 1326 out of a random sample of 5981 select jogging.

(a) Find a 95% confidence interval for p.
(b) Find an 80% confidence interval for p.

Answers: point estimate is $\hat{p} = 0.222$

(a) The CI is $[0.211, 0.233]$

(b) The CI is $[0.215, 0.229]$

9. Let p_1 be the proportion of adult men that smoke and p_2 the proportion of adult women that smoke. In a random sample of 2950 men, 316 are smokers. In a random sample of 2435 women, 169 are smokers.

(a) Construct a 98% confidence interval for $p_1 - p_2$.
(b) Construct a 90% confidence interval for $p_1 - p_2$.

Answers: point estimates $\hat{p}_1 = 0.107$, $\hat{p}_2 = 0.0694$

(a) The CI is $[0.0198, 0.0555]$

(b) The CI is $[0.025, 0.0503]$