MA 486-1E (Statistics), Dr Chernov — Midterm test #2

Show your work. — Fri, Mar 7, 2003

1. (5 pts) Let $x_1, \dots, x_{21}$ and $y_1, \dots, y_{31}$ be two independent samples taken from $N(\mu_x, \sigma_x^2)$ and $N(\mu_y, \sigma_y^2)$, respectively. The test results are $\bar{x} = 5$, $s_x^2 = 100$, $\bar{y} = 7$ and $s_y^2 = 90$.

Test the hypothesis that these two normal distributions are equal. Let $\alpha = 10\%$. Remember that there are two parameters to take care of. Find explicitly the critical regions.

Partial answers: first we test the hypothesis $H_0 : \sigma_x^2 = \sigma_y^2$ against $H_1 : \sigma_x^2 \neq \sigma_y^2$. The critical region is

$$s_x^2/s_y^2 > F_{0.05}(20, 30) = 1.93 \quad \text{or} \quad s_y^2/s_x^2 > F_{0.05}(30, 20) = 2.04$$

We accept H_0.

Second, we test the hypothesis $H_0 : \mu_x = \mu_y$ against $H_1 : \mu_x \neq \mu_y$. The test statistic is

$$T = \frac{5 - 7}{\sqrt{\frac{20\times 100+30\times 90}{50}}\sqrt{\frac{1}{21}+\frac{1}{31}}} = -0.73$$

The critical region is $|T| > t_{0.05}(50) \approx z_{0.05} = 1.645$. We accept H_0.

2. (4 pts) Let X be $N(\mu, 16)$. We need to test the hypothesis $H_0 : \mu_0 = 13$ against $H_1 : \mu_1 = 12$. Let the critical region be $\bar{x} \leq c$. Find values for n and c so that $\alpha = 0.02$ and $K(\mu_1) = 0.999$. Do not forget to find the values of *both* n and c.

Answers:

$$n = \left[\frac{4(2.054 + 3.090)}{12 - 13}\right]^2 = 423.37$$

hence $n = 424$, and

$$c = 12 + 3.090 \frac{4}{\sqrt{424}} = 12.6$$

3. (4 pts) In a biology laboratory students test the Mendelian theory of inheritance using corn. The Mendelian theory claims that the frequencies of the four categories smooth and yellow, wrinkled and yellow, smooth and purple, and wrinkled and purple will occur in the ratio 9:3:3:1. If a student counted 557, 172, 181, and 50, respectively, for these four categories, would these data support the Mednelian theory? Let $\alpha = 0.1$.

Answer:

$$Q = \frac{(557-540)^2}{540} + \frac{(172-180)^2}{180} + \frac{(181-180)^2}{180} + \frac{(50-60)^2}{60} = 2.563$$

The critical region is $Q > \chi^2_{0.1}(3) = 6.251$, hence we accept H_0.

4. (4 pts) Let $x_1, \ldots, x_{21}$ be taken from $N(\mu, \sigma^2)$ to test $H_0 : \mu = 18$ against $H_1 : \mu \neq 18$. The test results are $\sum x_i = 304.5$ and $\sum x_i^2 = 6415.25$. Test the hypothesis at the 5% significance level?
[Bonus] What is the approximate p-value of the test?

Solution: first we compute

$$\bar{x} = 304.5/21 = 14.5$$

and

$$s_x^2 = (6415.25 - 304.5^2/21)/20 = 100$$

Then the test statistic is

$$T = \frac{14.5 - 18}{10/\sqrt{21}} = -1.604$$

The critical region is $|T| > t_{0.025}(20) = 2.086$. We accept H_0.

The p-value is between 0.1 and 0.2.

5. (4 pts) Let $x_1, \ldots, x_{28}$ be taken from $N(\mu, \sigma^2)$ to test $H_0 : \sigma^2 = 15$ against $H_1 : \sigma^2 \neq 15$. The test results are $\bar{x} = 155$ and $s^2 = 23.1$.
Test the hypothesis at the 2% significance level.
[Bonus] What can you say about the p-value of the test?

Answer: the test statistic is

$$\chi^2 = \frac{27 \times 23.1}{15} = 41.58$$

The critical region is

$$\chi^2 < \chi^2_{0.99}(27) = 12.88 \quad \text{or} \quad \chi^2 > \chi^2_{0.01}(27) = 46.96$$

We accept H_0.

The p-value is between 0.05 and 0.1.

6. (5 pts) Let p_1 and p_2 be the proportions of babies with a low birth weight in the countries A and B, respectively. We shall test the hypothesis $p_1 = p_2$ against $p_1 > p_2$. In random samples of sizes $n_1 = 800$ and $n_2 = 1200$ babies in the two countries, $y_1 = 94$ and $y_2 = 117$ babies were found to have low birth weight. Test the hypothesis at the significance level $\alpha = 0.05$.

[Bonus] By using normal approximation, find the p-value of the test.

Solution: first, we estimate

$$\hat{p} = \frac{94 + 117}{800 + 1200} = 0.1055$$

Now the test statistic is

$$Z = \frac{94/800 - 117/1200}{\sqrt{0.1055 \times 0.8945 \times (1/800 + 1/1200)}} = 1.426$$

The critical region is $Z > z_{0.05} = 1.645$. We accept H_0.

The p-value is $1 - \Phi(1.426) = 0.0778$.

7. (4 pts) Let $x_1, \ldots, x_{100}$ be taken from $N(\mu, 9)$. To test $H_0 : \mu = 26$ against $H_1 : \mu < 26$ let the critical region be $\bar{x} < 25.1$. Compute α. Write down a formula for $K(\mu)$.

[Bonus] Compute $K(24.0)$, $K(24.5)$, $K(25.0)$, $K(25.5)$. Sketch the graph of the power function.

Answers:

$$\alpha = \Phi\left(\frac{25.1 - 26}{\sqrt{9}/\sqrt{100}}\right) = 0.0013$$

and

$$K(\mu) = \Phi\left(\frac{25.1 - \mu}{0.3}\right)$$

Answer to bonus question: $K(24.0) \approx 1$, $K(24.5) = 0.9772$, $K(25.0) = 0.6293$, and $K(25.5) = 0.0918$. The graph is not shown here, sorry.